

## BASIC RULES FOR PROPOSITIONAL LOGIC

### Notation:

$p, q, r, \dots$  are statement variables.

Truth functional connectives are  $\sim$  "not",  $\vee$  "or",  $\cdot$  "and",  $\supset$  "implies",  $\equiv$  "is equivalent to".

The symbol  $\vdash$  means "therefore".

### Introduce line:

One or two lines in a proof justify introducing a new line by inference.

Modus Ponens	$(p \supset q), p \vdash q$
Modus Tollens	$(p \supset q), \sim q \vdash \sim p$
Hypothetical Syllogism	$(p \supset q), (q \supset r) \vdash p \supset r$
Disjunctive Syllogism	$(p \vee q), \sim p \vdash q$
Constructive Dilemma	$[(p \supset q) \cdot (r \supset s)], (p \vee r) \vdash q \vee s$
Destructive Dilemma	$[(p \supset q) \cdot (r \supset s)], (\sim q \vee \sim s) \vdash \sim p \vee \sim r$
Bidirectional Dilemma	$[(p \supset q) \cdot (r \supset s)], (p \vee \sim s) \vdash q \vee \sim r$
Simplification	$p \cdot q \vdash p$
Conjunction	$p, q \vdash p \cdot q$
Addition	$p \vdash p \vee q$
Composition	$(p \supset q) \cdot (p \supset r) \vdash p \supset (q \cdot r)$

### Replace expression:

All or part of a line may be replaced by its equivalent.

DeMorgan nand	$\sim(p \cdot q) \equiv (\sim p \vee \sim q)$
DeMorgan nor	$\sim(p \vee q) \equiv (\sim p \cdot \sim q)$
Disjunctive Commutation	$(p \vee q) \equiv q \vee p$
Conjunctive Commutation	$(p \cdot q) \equiv (q \cdot p)$
Biconditional Commutation	$(p \equiv q) \equiv (q \equiv p)$
Association (or)	$[p \vee (q \vee r)] \equiv [(p \vee q) \vee r]$
Association (and)	$[p \cdot (q \cdot r)] \equiv [(p \cdot q) \cdot r]$
Distribution (or over and)	$[p \vee (q \cdot r)] \equiv [(p \vee q) \cdot (p \vee r)]$
Distribution (and over or)	$[p \cdot (q \vee r)] \equiv [(p \cdot q) \vee (p \cdot r)]$
Double Negation	$p \equiv \sim\sim p$
Transposition	$(p \supset q) \equiv (\sim q \supset p)$
Material Implication	$(p \supset q) \equiv (\sim p \vee q)$
Material Equivalence (implies)	$(p \equiv q) \equiv [(p \supset q) \cdot (q \supset p)]$
Material Equivalence (or)	$(p \equiv q) \equiv [(p \vee \sim q) \cdot (\sim p \vee q)]$
Material Equivalence (and)	$(p \equiv q) \equiv [(p \cdot q) \vee (\sim p \cdot \sim q)]$
Exportation/Importation	$[(p \cdot q) \supset r] \equiv [p \supset (q \supset r)]$
Tautology (or)	$p \equiv (p \vee p)$
Tautology (and)	$p \equiv (p \cdot p)$
Tertium Non Datur	$p \vee \sim p$
Contradiction	$\sim(p \cdot \sim p)$

**Special Arguments:**

Conditional argument only applies when conclusion is an implication. Indirect argument is unrestricted.

Conditional Argument

$$[(Premises) \vdash p \supset q] \equiv [(Premises \cdot p) \vdash q]$$

Indirect Argument

$$[(Premises) \vdash p] \equiv [(Premises \cdot \neg p) \vdash (q \cdot \sim q)]$$

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