Space curve $\mathbf{r}(t) = x(t)\,\hat{\imath} + y(t)\,\hat{\jmath} + z(t)\,\hat{k}$

Velocity $\mathbf{v}(t) = \mathbf{r}'(t)$ (defn)

Speed $|\mathbf{v}(t)| = s'(t)$ (defn)

Distance along curve $s(t) = \int_{t_0}^{t} |\mathbf{v}(t)|\,dt$ (defn)

Unit tangent $\mathbf{T} = \dfrac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} = \mathbf{r}'(s)$ ← (defn)

Unit normal $\mathbf{N} = \dfrac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} = \dfrac{1}{\kappa}\,\mathbf{T}'(s)$ ← (defn)

Unit binormal $\mathbf{B} = \mathbf{T} \times \mathbf{N}$ (defn)

Curvature $\kappa = \dfrac{|\mathbf{T}'(t)|}{|\mathbf{v}(t)|} = |\mathbf{T}'(s)|$ ← (defn)

Torsion $\tau = -\mathbf{B}'(s) \cdot \mathbf{N}$ (defn)

Practical Calculations

$$T = \frac{\vec{v}}{|\vec{v}|}$$

$$N = \frac{T'(t)}{|T'(t)|}$$

$$B = T \times N$$

$$\kappa = \frac{|\vec{v} \times \dot{\vec{v}}|}{|\vec{v}|^3}$$

"$\cdot$" is time derivative

$$\tau = \frac{\begin{vmatrix} \dot{x} & \dot{y} & \dot{z} \\ \ddot{x} & \ddot{y} & \ddot{z} \\ \dddot{x} & \dddot{y} & \dddot{z} \end{vmatrix}}{|\vec{v} \times \dot{\vec{v}}|^2}$$

$$a = \dot{\vec{v}} = a_T T + a_N N$$

$$a_T = \frac{d}{dt}|\vec{v}| \qquad a_N = \kappa [s'(t)]^2 = \kappa |\vec{v}|^2$$