Metric Units - Overview

The philosophy of the metric system is to define all units required by science and engineering in terms of a minimal set of **basic units**, which are themselves based on fundamental constants of the universe. The size of the set of basic units is dictated by the number of independent natural phenomena that must be described. Presently, there are seven basic units addressing, respectively, time, length, mass, amount of substance, heat, electricity, and light.

A key consideration in selecting a basic unit is its metrological utility. For example, it may seem like a good idea to define the unit of electrical charge as that carried by a certain quantity of electrons, since the charge on one electron is believed to be a fixed, universal quantity, and a collection of electrons could be specified with the perfect accuracy of an exact number. It turns out that although the number of electrons could be given exactly, the charge on one electron is not known or presently knowable with sufficient accuracy to make this approach preferable to one based on measuring electrical current. Current measurements depend on magnetic effects, which intrinsically admit a more accurate measurement than the charge on an electron. Of the seven basic phenomena, time can be measured to 1 part in 10^{14}, and is the most accurately known. Light intensity is the least accurately known at 1 part in 10^{3}. Although the amount of substance seems to be specifiable exactly with a pure number of elementary entities, the problem above with electrons repeats itself. Ultimately, the share of each elementary entity has to be measured, and this is subject to metrological inaccuracy. For example, weights are typically accurate to about 1 part in 10^{8} using the state-of-the-art gravimetric methods.

Using the basic units, a larger class of **derived units** can be constructed. Derivation is based on three methods: (i) definitional, (ii) geometric formulas, and (iii) physical formulas. The first two methods are, of course, exact. The third requires acceptance of the physical theory underlying the formula. An earlier category of **supplementary units** consisting of the radian and steradian have been reclassified as derived.

Ironically, the definitions of some of the basic units require consideration of phenomena which are the subject of derived units. For example, the definition of the unit of electric current (ampere) requires the notion of force, measured by a derived unit (newton). This is not logically objectionable, since there is no circularity in the definitions requiring this approach.

Units named after an individual are not capitalized if written out, but the abbreviation is always capitalized. Units not named after an individual are lower case in both full and abbreviated form.

Metric Units - Basic

Basic units are in boldface. Derived units required to define the basic units are also reviewed.

(1) **Length** - one **meter** is the distance that light travels in $\frac{1}{299792458}$ second. Light travels at 299 792 458 meters per second. When the first Convention du Metre was called in 1875, this velocity was poorly known, although astronomical observations suggested it was finite. Albert A. Michelson, the American physicist, measured the speed of light, universally denoted by c, in the 1880's using terrestrial apparatus.

(2) **Time** - one **second** is the duration of 9 192 631 770 periods of the radiation corresponding to the ground state hyperfine transition of cesium-133. This is not visible light - the wavelength is 32.61 millimeters. The atomic clocks around the world use this as a basis for timekeeping.

(3) **Mass** - the prototype **kilogram** is kept in Sevres, France by the BIPM. It is a 90% platinum/10% iridium cylinder. Gravimetric methods are sufficiently accurate to allow secondary prototypes to be made that conform to the original. Any body having the same inertial or gravitational mass as the prototype cylinder is deemed to have mass equal to one kilogram.

(4) Solid Angle - one steradian is the solid angle subtended by $\frac{1}{4\pi}$ of the surface of a sphere. Solid angle is a derived unit based on the geometric formula for the area of a sphere of radius r: $A = 4\pi r^2$. It is needed to define the basic unit of luminous intensity.

(5) Force - The three boxed basic units define force through a law of physics, namely, Newton's second law ($F = ma$) which asserts that the acceleration of an object is directly proportional to the force applied to it. Actually a little more is implied regarding the collinearity of the force and acceleration. The constant of proportionality is mass. A newton is the force required to accelerate a mass of one kilogram one meter per second per second. Although force is a derived unit, it is needed to define the unit of electric current and also energy, hence power, and ultimately luminous intensity.

(6) **Amount of Substance** - One **mole** (formerly gram molecular weight) of a substance is the amount of it such that the number of elementary entities present is the same as in exactly 0.012 kilograms of pure carbon-12. This number, incidentally, is Avogadro's constant, or approximately 6.022×10^{23} particles. It is essential to specify, when talking about moles, just what the elementary entities are. For example, one mole of oxygen molecules is different than one mole of oxygen atoms, because the molecules each consist of two atoms. Mathematicians would regard the mole as a superfluous unit, since mass could be used to specify the amount of substance, and the set of basic units could be reduced by one. However, chemists like the idea, since it simplifies the study of chemical combinations. Relative mass is already accounted for in the mole.

(7) Work - A force of one newton acting through a distance of one meter does one joule of work. It is assumed that the force and displacement are parallel. By the way, James Prescott Joule was a Scottish physicist and his name was pronounced "jool". Work and energy are the same thing.

(8) Power - the transfer of one joule per second is defined to be a power of one watt. James Watt, yet another Scot, invented the modern steam engine. Power is the rate of doing work or the rate of transferring energy.

(9) Pressure - by definition, one newton acting on one square meter constitutes a pascal.

(10) **Luminous Intensity** - Luminosity is ultimately the number of photons per unit of time that move through a given solid angle as they are emitted from a point source. Energy is used as a proxy for the number of photons. By Planck's Law, the energy of a photon of frequency ν is $h\nu$. By fixing the frequency of standard radiation at 540×10^{12} cycles per second, and the power (hence energy) of such radiation at $\frac{1}{683}$ watt per steradian, a certain number of photons per unit of time per steradian is implied. This amount is one **candela**. Luminosity is the least accurately measured of the phenomena underlying the seven basic units.

(11) **Current** - Electrical phenomena enter the SI through the unit for current, the **ampere**. The

following thought experiment is governed by an equation of Maxwell (Scottish, of course). According to this equation, $\nabla \times B = \frac{1}{c^2}(\frac{J}{\epsilon_0} + \frac{\partial E}{\partial t})$, where B is the magnetic field, J is the current density, E is the electric field, c is, as usual, the speed of light, ϵ_0 is the permittivity of the vacuum, and t is time. Suppose that two infinitely long, infinitesimally thin conductors are parallel and carrying the same current. If they are one meter apart and exert a force of 2 x 10^{-7} newtons per meter of wire on each other, they are said to be carrying a current of one ampere. The strange choice of the constant force has a historical origin. In the old MKS system of metric measurement (the precursor to SI), the product $4\pi\epsilon_0 c^2$ was exactly 10^7. A distance r from one wire the circulation of the magnetic field is (by symmetry) $2\pi rB$. By applying Stokes' theorem to Maxwell's equation, this must equal $\oint B \cdot dl = \iint \nabla \times B \cdot ndA = \frac{1}{\epsilon_0 c^2}\iint J \cdot ndA = \frac{I}{\epsilon_0 c^2}$, and it follows that $B = \frac{I}{2\pi\epsilon_0 c^2 r} = \frac{2I}{4\pi\epsilon_0 c^2 r} = (2x10^{-7})\frac{I}{r}$, which is Ampere's Law. Note that the ideal nature of this experiment causes the infinite current density integrated over an infinitesimal area to be just the charge per unit of time, or I. It is worth remembering that the force per unit length on a uniform wire carrying current I in a constant magnetic field B is simply $I \times B$. So one meter away from a wire carrying a current of one ampere the B field is $2x10^{-7}$ $[\frac{N}{A-m}]$, and a wire carrying the identical current would feel a force equal to $2x10^{-7}$ $[\frac{N}{m}]$.

(12) **Temperature** - heat is energy, so we already can assign a unit to heat. The manifestation of heat energy, namely temperature, which is critical to the Second Law of Thermodynamics, requires its own unit. Temperature defines the direction for transfer of energy in the form of heat. Thermodynamic temperature is counted from absolute zero - the theoretical temperature at which the kinetic energy of molecules comprising a substance is at a minumum. The scale is arbitrary, but fixing the triple point of pure water at 273.16 units above absolute zero defines the **kelvin**. The triple point is the temperature/pressure condition in which all three phases of water are in equilibrium. Implicit in the triple point is the ambient pressure condition, which is taken to be 611.2 pascals. For practical purposes, a range of sixteen fixed thermodynamic temperatures amplifies the scale. For example, pure aluminum freezes at 933.473 kelvins by international convention.

Metric Prefixes

10^{24}	yotta	Y
10^{21}	zetta	Z
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{0}	base unit	
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a
10^{-21}	zepto	z
10^{-24}	yocto	y

Notes:

(1) The historical prefixes for multipliers other than those of the form 10^{3n} as above are now discouraged. These include centi, deci, deka, and hecto.

(2) Allowed prefixes are to be appended to SI units with no hyphen.

(3) Abbreviations are capitalized for orders of magnitude 10^6 and above. Abbreviations are latin letters with the sole exception of μ for micro.

(4) The convention is to give units in square brackets, for example 2.056 [m].

acceleration	meter per second per second
amount	mole
angle, solid	steradian
angle, plane	radian
angular velocity	radian per second
area	square meter
concentration	mole per cubic meter
density, mass	kilogram per cubic meter
electrical resistance	ohm
electrical conductance	siemens
electrical inductance	henry
electrical capacitance	farad
electrical current density	ampere per square meter
electrical field strength	volt per meter
electrical current	ampere
electrical charge	coulomb
electromotive force	volt
electrical resistivity	ohm-meter
energy	joule
flow, volumetric	cubic meter per second
force	newton
frequency	hertz
length	meter
luminance	candela per square meter
illuminance	lux
luminous intensity	candela
luminous flux	lumen
magnetic moment	joule per tesla
magnetomotive force	ampere
magnetic flux	weber
magnetic field strength	ampere per meter
magnetic flux density	tesla
mass	kilogram
molality	mole per kilogram of solvent
moment of inertia	$meter^4$
power	watt
power density	watt per square meter
power level difference	bel, neper
pressure	pascal
section modulus	cubic meter
stress	pascal
temperature	kelvin
thermal conductivity	watt per kelvin-meter
time	second
torque	newton-meter
velocity	meter per second
viscosity, dynamic	pascal-second
viscosity, kinematic	square meter per second
volume	cubic meter

Metric Units - Derived

- *GEOMETRIC*

radian	[rad]	angle subtended by $\frac{1}{2\pi}$ part of the circumference of a circle
steradian	[sr]	solid angle (see basic units)

- *FREQUENCY*

hertz	[Hz]	frequency of periodic phenomena, one cycle per second
becquerel	[Bq]	average frequency of non-periodic nuclear phenomenon, one per second

- *MECHANICAL*

newton	[N]	force (see basic units)
joule	[J]	work or energy, (see basic units)
pascal	[Pa]	pressure (see basic units)
watt	[W]	power (see basic units)

- *ELECTRICAL*

coulomb	[C]	electrical charge equal to one ampere flowing for one second
volt	[V]	potential difference of one joule per coulomb
farad	[F]	capacitance of one coulomb per volt
henry	[H]	inductance of one volt per ampere per second
ohm	[Ω]	resistance of one volt per ampere
siemens	[S]	conductance of one reciprocal ohm

- *MAGNETIC*

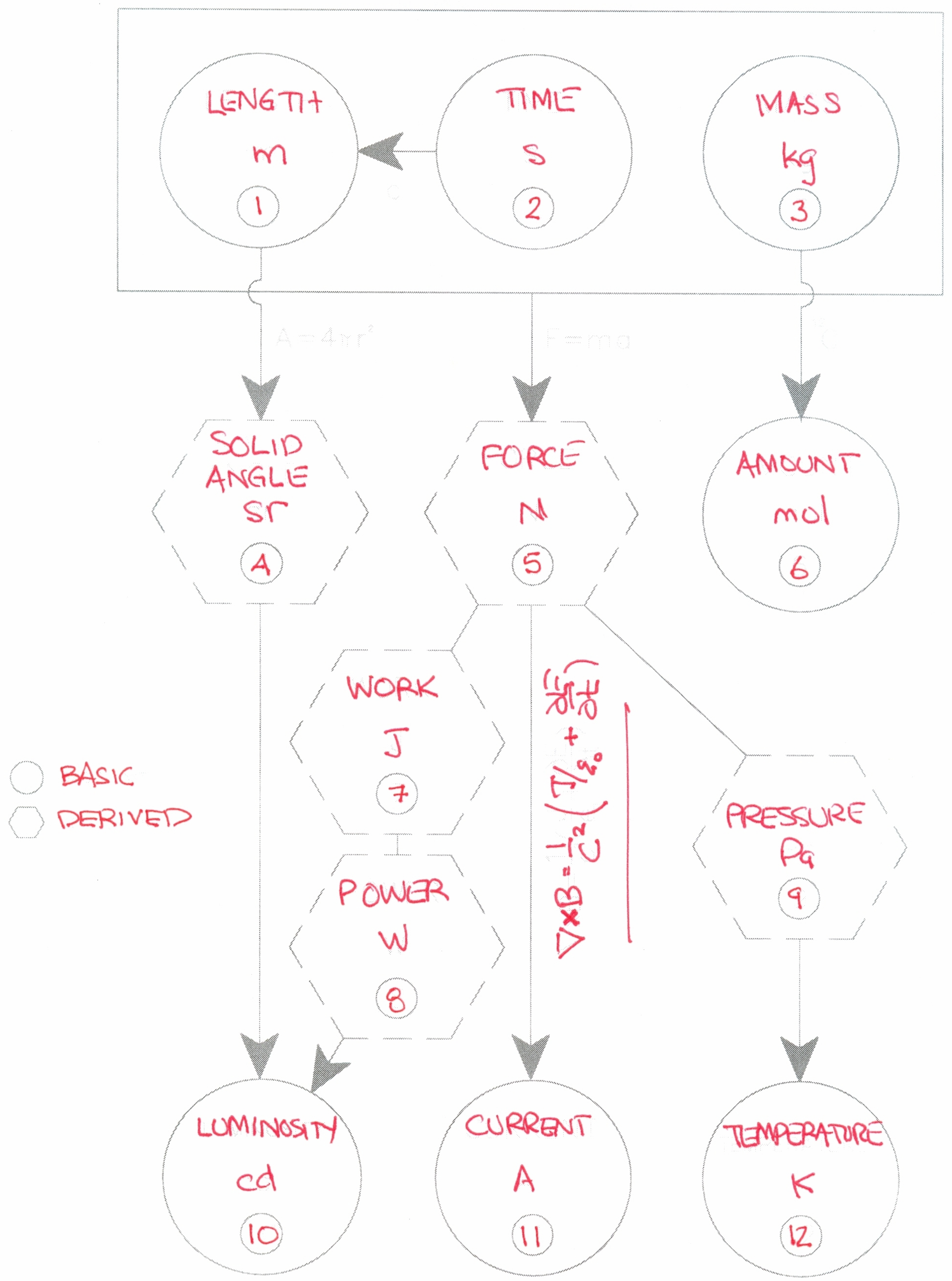
weber	[Wb]	flux of one volt-second
tesla	[T]	flux density of one weber per square meter

- *LIGHT*

lumen	[lm]	flux of one candela-steradian
lux	[lx]	illuminance of one lumen per square meter

- *RADIOLOGICAL*

gray	[Gy]	absorbed radiation equal to one joule per kilogram of tissue
sievert	[Sv]	dose equivalent equal to one joule of standard radiation per kg of tissue


LENGTH
m
1
TIME
s
2
MASS
kg
3
A=4πr²
F=ma
SOLID ANGLE
sr
4
FORCE
N
5
AMOUNT
mol
6
WORK
J
7
POWER
W
8
PRESSURE
Pa
9
$\nabla \times B = \frac{1}{c^2}\left(\frac{J}{\varepsilon_0} + \frac{\partial E}{\partial t}\right)$
BASIC
DERIVED
LUMINOSITY
cd
10
CURRENT
A
11
TEMPERATURE
K
12