MODERN ANALYSIS 1 - FALL 2024 - PROBLEM SET 4 - due 10/3

1) If $s_n \leq t_n$ for all n greater than some fixed N, show $\limsup_{n\to\infty}\langle s_n \rangle \leq \limsup_{n\to\infty}\langle t_n \rangle$ and $\liminf\langle s_n \rangle \leq \liminf\langle t_n \rangle$

2) Show that for any $\varepsilon > 0$ there are only finitely many values $s_n \geq \limsup_{n\to\infty}\langle s_n \rangle + \varepsilon$

3) Show $\lim_{n\to\infty} \sqrt[n]{n} = 1$ using L'Hôpital's Theorem

4) Let $a_{n+1} = \sqrt{\pi a_n}$ where $a_1 = 1$. Prove or disprove whether $\lim_{n\to\infty} a_n$ exists and if it does, find it.