MODERN ANALYSIS 1 - FALL 2024 - PROBLEM SET 2 - due 9/3 More hints in class.

1) Use induction to prove $\sum_{k=1}^{n} \frac{1}{k^2} = \frac{n(n+1)(2n+1)}{6}$

2) Now use induction to show that a countable union of countable sets is countable. In symbols, if $card A_i = \aleph_0 \ \forall i \in \mathbb{N}$, then $card \bigcup_{i=1}^{\infty} A_i = \aleph_0$ Hint: Base case argument works for induction step.

3) Prove that $|\mathbf{x}+\mathbf{y}|^2 + |\mathbf{x}-\mathbf{y}|^2 = 2|\mathbf{x}|^2 + 2|\mathbf{y}|^2$ for $\mathbf{x}, \mathbf{y} \in \mathbb{R}^k$ and interpret geometrically as a statement about parallelograms

4) Under what conditions does equality hold in the Cauchy-Schwarz-Bunyakowsky inequality?