

MODERN ANALYSIS 1 - FALL 2024 - PROBLEM SET 1 - due 8/27

1) If  $r \in \mathbb{Q}$  and  $r \neq 0$  and  $x \notin \mathbb{Q}$ , show  $r+x \notin \mathbb{Q}$  and  $rx \notin \mathbb{Q}$

2) The number  $\alpha = \sqrt{2}^{\sqrt{2}}$  is irrational (in fact it is transcendental). Is it possible for a power of  $\alpha$  to be rational?

3) For  $x, y \in \mathbb{R}$ , show that  $|x - y| \geq ||x| - |y||$

4) Propose a reasonable definition of the cosine of the angle between two four dimensional vectors.