Solution to Homework #3 grad problem:

We can reduce the five TFO's to two, and then merge them into one as follows:

1) $A \Leftrightarrow B$ can always be expressed as $(A \Rightarrow B) \cdot (B \Rightarrow A)$, so eliminate "$\Leftrightarrow$"

2) $A \Rightarrow B$ is equivalent to $\sim A \lor B$, so eliminate "$\Rightarrow$" by material equivalence

3) $A \lor B$ is equivalent to $\sim(\sim A \cdot \sim B)$ by DeMorgan, so eliminate "v"

4) Now define $A \uparrow B$ as $\sim(A \cdot B)$. This is the NAND operator.

5) $(A \uparrow B) \uparrow (A \uparrow B)$ is the same as $\sim(\sim(A \cdot B) \cdot \sim(A \cdot B)) = \sim(\sim(A \cdot B) = A \cdot B$, so we can dispense with "•"

6) Finally, $(A \uparrow A)$ is equivalent to $\sim A$, so we can drop "~"

7) Note that there is no way to express • in terms of ~ or vice versa, so we are stuck with ↑ (NAND) or ↓ (NOR), which works the same way.

8) Also $(A \uparrow A) \uparrow (B \uparrow B) = \sim(\sim(A \cdot A) \cdot \sim(B \cdot B)) = \sim(\sim A \cdot \sim B) = A \lor B$ by DeMorgan

9) A standard element on a semiconductor chip is a cluster of four "NAND gates" each having a common ground, two inputs and one output. With enough of these you can create any logic circuit. We'll try that in the next homework.