SYMBOLIC LOGIC - EXAM 1 - Solutions

Provide symbolizations of the following sequences of sentences (presented as arguments).
Please use the suggested abbreviations.

1) (A,B,C,E) Either Argentina does not join the alliance or Brazil boycotts it, but if Argentina joins the alliance then Chile boycotts it. If Brazil boycotts the alliance, then if Chile boycotts it then Ecuador will boycott it. Therefore, if Argentina joins the alliance, then Ecuador will boycott it.

A = Argentina joins / B = Brazil boycotts / C = Chile boycotts / E = Ecuador boycotts...could treat boycott as not joining, and express everything in terms of joining, but only A is present both positive and negated

$((\sim A \lor B) \land (A \supset C))$
$(B \supset (C \supset E))$
$\therefore (A \supset E)$

2) (T,E,A,L) If you plant tulips, then your garden will bloom early, and if you plant asters, then your garden will bloom late. So if you plant either tulips or asters, your garden will bloom either early or late.

$((T \supset E) \land (A \supset L))$
$\therefore ((T \lor A) \supset (E \lor L))$

3) (T,E,A,L) If you plant tulips, then your garden will bloom early, and if you plant asters, then your garden will bloom late. So if you plant both tulips and asters, your garden will bloom both early and late.

$((T \supset E) \land (A \supset L))$
$\therefore ((T \land A) \supset (E \land L))$

4) (A,B,C,E,P,V,N,U) If either Argentina or Brazil joins the alliance, then if either Chile or Ecuador boycotts it, then although Peru does not boycott it, Venezuela boycotts it. If either Peru or Nicaragua does not boycott it, then Uruguay will join the alliance. Therefore, if Argentina joins the alliance, then if Chile boycotts it, then Uruguay will join the alliance.

A = Argentina joins / B = Brazil joins / C = Chile boycotts / E = Ecuador boycotts / P = Peru does not boycott / V = Venezuela boycotts / N = Nicaragua does not boycott / U = Uruguay joins...could treat boycott as not joining, and express everything in terms of joining,

but no constant is present as both positive and negated

$((A \lor B) \supset ((C \lor E) \supset (P \land V)))$
$((P \lor N) \supset U)$
$\therefore (A \supset (C \supset U))$

5) (P,C,N,R,E,T) If he enters the primary, then if he campaigns vigorously then he wins the nomination. If he wins the nomination and receives the support of the party regulars, then he will be elected. If he takes the party platform seriously, then although he receives the support of the party regulars, he will not be elected. Therefore, if he enters the primary, then if he campaigns vigorously, then he does not take the party platform seriously.

$(P \supset (C \supset N))$
$((N \land R) \supset E)$
$(T \supset (R \land {\sim}E))$
$\therefore (P \supset (C \supset {\sim}T))$

*We use the short truth table method and try to engineer true premises implying a false conclusion with suitable truth values of the component statements. If this can be arranged, the argument is invalid. If it cannot be done, the argument is valid. When it can't be done, one or more of the premises is forced to be false as well.*

6) If Ed wins first prize, then either Fred wins second prize or George is disappointed. Fred does not win second prize. Therefore, if George is disappointed, then Ed does not win first prize.

E = Ed wins / F = Fred wins / G = George disappointed
The only way the conclusion can be false is if the antecedent of the material implication is true and the consequent false. This determines the truth values of E and G. We may choose F to be false and then the premises are both true and conclusion false, so **invalid**.

| $E$ | $\supset$ | $(F$ | $\lor$ | $G)$ | $\sim$ | $F$ | $G$ | $\supset$ | $\sim$ | $E$ |
|---|---|---|---|---|---|---|---|---|---|---|
| t | T | f | t | t | T | f | t | F | f | t |

7) If the weather is warm and the sky is clear, then either we go swimming or we go boating. It is not the case that if we do not go swimming then the sky is not clear. Therefore, either the weather is warm or we go boating.

W = weather warm / C = sky clear / S = go swimming / B = go boating. Turn the conditional in the second premise into its contrapositive for clarity: It is not the case that if the sky is clear then we go swimming. For the conclusory disjunction to be false, both

disjuncts must be false, so we take W and B to be false. We want the contrapositve to be false so that its negation (the second premise) is true. So we take C true and S false. This leaves a false conjunction implying a false disjunction in the first premise, which is true. So the argument is **invalid**.

$$(W \ \wedge \ C) \ \supset \ (S \ \vee \ B) \ \sim \ (C \ \supset \ S) \ (W \ \vee \ B)$$
$$f \quad f \quad t \quad T \quad f \quad f \quad f \quad T \quad t \quad f \quad f \quad f \quad F \quad f$$

8) If the seed catalog is correct, then if the seeds are planted in April then the flowers bloom in July. The flowers do not bloom in July. Therefore, if the seeds are not planted in April, then the seed catalog is not correct.

S = catalog correct / A = plant in April / J = bloom in July. For the conclusion to be false, A must be false and S true. To make the second premise true, J must be false. Then the first premise is true and we see the argument is **invalid**.

$$S \ \supset \ (A \ \supset \ J) \ \sim \ J \ \sim \ A \ \supset \ \sim \ S$$
$$t \quad T \quad f \quad t \quad f \quad T \quad f \quad t \quad f \quad F \quad f \quad t$$


9) If Alice is elected class president, then either Betty is elected vice-president or Carol is elected treasurer. Carol is not elected treasurer. Therefore, if Betty is not elected vice-president, then Alice is not elected class president.

A = Alice elected / B = Betty elected / C = Carol elected For the conclusion to be false, B must be false and A true. Now the first premise will be true only if C is true. But then the second premise is forced to be false. All of the truth value assignments are forced once the conclusion is assumed false, so we see one premise is necessarily false. This means the argument is **valid**.

$$A \ \supset \ (B \ \vee \ C) \ \sim \ C \ \sim \ B \ \supset \ \sim \ A$$
$$t \quad T \quad f \quad t \quad t \quad F \quad t \quad t \quad f \quad F \quad f \quad t$$

10) If the supply of silver remains constant and the use of silver increases, then the price of silver rises. If an increase in the use of silver implies that the price of silver rises, then there will be a windfall for speculators. The supply of silver remains constant. Therefore, there will be a windfall for speculators.

C = supply constant / U = use increases / P = price rises / W = windfall to speculators. Let W be false. Set the premise C true. For the second premise to be true, since the main consequent is already false, we must have the implication U implies P false. The only way

this can happen is for U to be true and P false. Now the truth values of C, U, and P in the first premise are fixed. We see this forces the implication in the first premise to be false. So assuming a false conclusion forces a premise to be false. Hence the argument is **valid**.

$$(C \quad \wedge \quad U) \quad \supset \quad P \quad (U \quad \supset \quad P) \quad \supset \quad W \quad C \quad W$$

$$t \quad t \quad t \quad F \quad f \quad t \quad f \quad f \quad T \quad f \quad T \quad F$$