ELEMENTARY CALCULUS 1 - FALL 2024 - EXAM 4B - Solutions

45 minutes - all honorable references permitted. Each question equal weight. Total 50 points = half of Exam 3.

1) A bus will hold 60 people. The number x of people per trip who use the bus is related to the fare charged p (in dollars) by the law $p = [3 - \frac{x}{40}]^2$. Find the number of people per trip that will maximize the revenue for the bus company and what fare they should charge. Should the bus company revise its fare policy?

Revenue per trip is $R(x) = xp = x[3 - \frac{x}{40}]^2$. Marginal revenue is after simplification $R'(x) = 9 - \frac{3}{10}x + \frac{3}{1600}x^2$. Setting this equal to zero and solving, we have $x = 40$ or 120. We discard 120 as the bus capacity is only 60. Checking the second derivative we get $R''(40) = -\frac{3}{10} + \frac{3}{800}(40) = -0.15$. So the concavity is down and $x = 40$ gives a maximum. With this pricing function they should charge $\$4$ per trip. Since the capacity of the bus is much greater, the bus company should try to push the optimum occupancy toward 60.

2) A cube of ice is initially one inch on a side. The ice melts evenly all around so that any side x loses 0.05 inches per minute. What is the rate of weight loss of the cube when it is half an inch on each side? When is the cube completely melted? Hints: Volume equals the side cubed, weight of ice is 57 lbs per cubic foot.

Let V be the volume of the cube and x be side, then $V = x^3$. Now let W be the weight of the shrinking cube. Volume times weight per unit of volume is total weight, so $W = 0.033V = 0.033x^3$ since 57 pounds per cubic foot is $\frac{57}{1728} = 0.033$ pounds per cubic inch. Now differentiate both sides with respect to time:
$\frac{dW}{dt} = (0.033)(3x^2)\frac{dx}{dt} = (0.099)x^2\frac{dx}{dt}$. Plugging in $x = 0.5$ and $\frac{dx}{dt} = -0.05$, we get
$\frac{dW}{dt} = (0.099)(0.5)^2(-0.05) = -0.00124$ pounds per minute. The original cube was one inch on a side and it is losing $\frac{1}{20}$ inch per minute, so after 20 minutes it will have completely melted.

3) An artist could sell fifty prints at \$100 each. From past experience she knows she can sell ten more for each dollar price reduction. What should her price be to maximize her revenue?

Let x be the dollar price reduction from $\$100$, so her price would be $100 - x$ per print. Then her expected unit sales would be $50 + 10x$. Her dollar sales would be $R(x) = (100 - x)(50 + 10x)$. This simplifies to $R(x) = -10x^2 + 950x + 5000$. Setting the marginal revenue equal to zero we get $R'(x) = -20x + 950 = 0$, or $x = 47.5$. Checking the second *derivative,* $R''(x) = -20$*, so this is a maximum. Her price should be* $100 - 47.5 = 52.5$ *each, and this gives a best possible revenue of* $\$27,562.50$*.*