

## ELEMENTARY CALCULUS 1 - FALL 2024 - EXAM 3B - Solutions

1) Find  $f'(x)$  if  $f(x) = \frac{e^{2x}(x^3 + 2x + 1)}{x^2 + 4x}$

*Simplified, it is*  $\frac{e^{2x}}{x^2(x+4)^2}(2x^5 + 9x^4 + 12x^3 + 16x^2 - 4)$

2) Find  $f'(x)$  if  $f(x) = \frac{e^x}{x \ln x}$

*Simplified it is*  $\frac{e^x(x \ln x - \ln x - 1)}{x^2(\ln x)^2}$

3) If the price  $p$  of a product is given by  $p = qe^{-q}$  where  $q$  is quantity sold, find marginal revenue  $R'(q)$

*Revenue is price times quantity, so  $R(q) = q \cdot qe^{-q} = q^2e^{-q}$ . Then  $R'(q) = 2qe^{-q} - q^2e^{-q}$*

4) If the cost  $C$  of a product is given by  $C(q) = .01q^2 + 7q + 50$  where  $q$  is quantity sold, find:

- (i) marginal cost
- (ii) average cost
- (iii) marginal average cost

*(i) marginal cost is  $C'(q) = .02q + 7$*

*(ii) average cost is  $\frac{C(q)}{q} = .01q + 7 + \frac{50}{q}$*

*(iii) marginal average cost is  $\left(\frac{C(q)}{q}\right)' = .01 - \frac{50}{q^2}$*

5) If  $f(x) = x^2 + e^x$  and  $g(x) = x^3 + 1$  find:

- (i)  $f(g(x))$
- (ii)  $[f(g(x))]'$

*(i) the composition is  $f(g(x)) = (x^3 + 1)^2 + e^{x^3+1}$*

*(ii) derivative of the composition is  $6x^2(x^3 + 1) + 3x^2e^{x^3+1} = 3x^2(2(x^3 + 1) + e^{x^3+1})$*