ELEMENTARY CALCULUS 1 - FALL 2024 - EXAM 2B - Solutions

1) A cliff is 1296 feet high. A stone rolls off the cliff and falls this entire distance.

(i) How long does it take to get to the bottom? $s(t) = 16t^2 = 1296$, so $t^2 = 81$ and $t = 9$ seconds

(ii) How fast is it going when it hits the ground (neglect air resistance) ? speed is $s'(t) = 32t$, so $s'(9) = 32 \cdot 9 = 288$ feet per second

(iii) What is its acceleration when it is halfway down? acceleration is $s''(t) = 32,$ feet per second per second, a constant

2) A business has a revenue function $R(x) = 30x^2 - 7x + 5$ in thousands of dollars where x is the number of items sold in thousands. The cost function for the business is $C(x) = 18x^2 + 5x - 10$ in thousands of dollars.

(i) What is the profit function? $P(x) = R(x) - C(x) = 12x^2 - 12x + 15$

(ii) What is the breakeven point for this business? graph $R(x)$ and $C(x)$ to see that profit is always positive, so no breakeven point / or use quadratic formula on $P(x) = 0$

(iii) What is the marginal cost at the production level $x = 10$? marginal cost is derivative of $C(x)$, so $C'(x) = 36x + 5$ and $C'(10) = \$365$ thousand dollars per thousand units

(iv) What is the marginal profit at production level $x = 40$? marginal profit is $P'(x) = 24x - 12$, so $P'(40) = \$948$ thousand dollars per thousand units

(v) What is the marginal profit at the breakeven point? not defined since there is no BE point

3) What is the derivative of $f(x) = (x^2 + 7x + 9) \cdot (x^2 + 2x + 3)$ multiply first to get $x^4 + 9x^3 + 26x^2 + 39x + 27$, then $f'(x) = 4x^3 + 27x^2 + 52x + 39$