ELEMENTARY CALCULUS 1 - FALL 2024 - EXAM 1B - Solutions

1) How long does it take to triple your money invested at 7.25% per year compounded continuously?

One dollar becomes three dollars at the given rate. So $3 = 1 \cdot e^{.0725t}$*. Take* ln *both sides:* $\ln 3 = 1.099 = 0.0725t$*. Solve for* t*. Then* $t = \frac{1.099}{0.0725} = 15.2$ *years*

2) If the price of a widget is $\$8.00$ and the cumulative cost function is $6.5x + 4500$ dollars where x is the number made, what is the breakeven point?

Revenue function is $8x$*. Set equal to cost function and solve for units produced. So* $8x = 6.5x + 4500$*, or* $1.5x = 4500$*. This gives* $x = 3000$ *units as breakeven production.*

3) If the linear variable cost to make 150 widgets is $\$2400$, what is the marginal cost?

Marginal cost is variable cost per unit. $MC = \frac{2400}{150} = 16$ *dollars per unit*

4) If the half-life of a radioactive material is 1000 years, what percent of its weight will be left after 1800 years?

From half-life formula, $0.5 = e^{-\lambda t}$*, where* t *is* 1000*. So* $0.5 = e^{-1000\lambda}$ *and taking the natural log,* $-0.69315 = -1000\lambda$*. Solve for the decay constant* $\lambda = \frac{0.6932}{1000} = 0.0006932$*. Take this* λ *back to the original decay equation* $Q(t) = Q(0)e^{-0.0006932(1800)}$*. So* $\frac{Q(t)}{Q(0)} = e^{-1.25} = 0.287$*, so* 28.7% *of the original weight is left.*

5) How much interest would a savings account earn in 7 years at 8.5% interest compounded quarterly?

$A = P(1 + \frac{0.085}{4})^{7 \cdot 4} = (1.02125)^{28} = \$1.80$ *per dollar invested, so the interest earned would be* 80 *cents*