FALL 2024 - CALCULUS 3 - TEST #3A - Solutions

$f(x,y)$ and $g(x,y)$ are functions with all partial derivatives of second order unless noted otherwise

True or false

T 1) $\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial y^2}$ sometimes

F 2) The sign of $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial y^2}$ are always the same

F 3) $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$ if these derivatives are continuous   not enough

F 4) $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$ is possible without continuity of these derivatives

F 5) $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$ if the first order partial derivatives are continuous   still not enough

T 6) $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$ if $f(x,y)$ and all its first and second order derivatives are continuous

Clairaut's (mixed partials) Theorem

F 7) If $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist at a point, then f is continuous there   need full differentiability

T 8) $f_{xy} = \frac{\partial^2 f}{\partial y \partial x}$ always

F 9) $D_{xy} f = \frac{\partial^2 f}{\partial x \partial y}$ always

T 10) The tangent plane to a surface at a point is perpendicular to the gradient there

F 11) $\Delta f(x,y,z) \approx \frac{\partial^2 f}{\partial x^2}\Delta x + \frac{\partial^2 f}{\partial y^2}\Delta y + \frac{\partial^2 f}{\partial z^2}\Delta z$ first partials not second

T 12) $(e^{x \sin y})_{xxy} = (e^{x \sin y})_{xyx}$ functions are all infinitely differentiable

T 13) ∇f has both vector and differential operator properties

T 14) $\nabla f \cdot u$ is a directional derivative in the direction of u

F 15) $f \nabla g = f_x g_x + f_y g_y + f_z g_z$ this would be $\nabla f \cdot \nabla g$

T 16) If u is not parallel to ∇f, $D_u f \leq |\nabla f|$ $\cos\theta \leq 1$

T 17) A gradient can be a direction vector for a tangent plane

T 18) A gradient can determine a normal line to a surface

F 19) If $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0$ everywhere, then $f = 0$ could be a constant

F 20) $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0$ at a point, then f is constant   could be a stationary point (peak or valley)

T 21) The gradient at a point on a level contour ($f(x,y) =$ constant) is always zero

F 22) $\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t}$ if x and y depend on t   ordinary derivatives

T 23) If $\mathbf{r}(t)$ is a plane curve, then $\frac{d}{dt} f(\mathbf{r}(t)) = \nabla f \cdot \mathbf{r}'(t)$ chain rule

T 24) The gradient obeys the product rule

F 25) The gradient represents the direction of maximum decrease of a function   increase