FALL 2024 - CALCULUS 3 - TEST 2A - Solutions

T 1) $\mathbf{r}'(t)$ is a vector tangent to the path vector $\mathbf{r}(t)$

F 2) The unit tangent vector $\mathbf{T}(t) = \mathbf{r}(t)/\|\mathbf{r}'(t)\|$ need $\mathbf{r}'(t)/\|\mathbf{r}'(t)\|$

F 3) The unit binormal vector $\mathbf{B}(t) = (\mathbf{r}'(t)/\|\mathbf{r}'(t)\|) \times \frac{d\mathbf{T}}{dt}$, where $\mathbf{T}(t)$ is the unit tangent vector need to unitize $\frac{d\mathbf{T}}{dt}$

T 4) The curvature of $\mathbf{r}(t)$ at a point is the reciprocal radius of the osculating circle at that point

F 5) The product of curvature and torsion is constant at any point of $\mathbf{r}(t)$ they are unrelated

F 6) Differential arc length $ds = \|\mathbf{r}(t)\|dt$ need $\|\mathbf{r}'(t)\|$

F 7) Torsion is a measure of the tendency for the unit principal normal to change along a curve binormal

T 8) If $\kappa(t) = 0$ for all t, then $\mathbf{N}(t)$ is undefined straight path

F 9) The rate of change of the binormal vector with distance has no component in the direction of the tangent vector both normal and tangential components

F 10) The derivative of a vector of constant length is zero direction still changing

F11) $\mathbf{r}(t) = \langle \cos t, \sin t, 1 \rangle$ describes a helix of constant pitch this is circle at $z = 1$

F 12) $\mathbf{r}(t) = \langle t, \cos t, \sin t \rangle$ describes a plane curve this is helix growing in x direction

T 13) $\mathbf{B}(t)$ for a plane curve is constant it is unit direction vector for the plane

F 14) $\mathbf{B}(t)$ for a helix is constant

T 15) τ for a helix is constant also curvature

F 16) Ballistic trajectories are for both self-propelled and non-self-propelled objects only non-self-propelled

T 17) The binormal vector to a curve is a direction vector for the osculating plane $\mathbf{N}$ and $\mathbf{T}$ are in the osculating plane

F 18) $\mathbf{r}''(t)$ is always perpendicular to $\mathbf{r}'(t)$ can have tangential acceleration

F 19) $\|\mathbf{r}'(t)\|$ is always perpendicular to $\|\mathbf{r}(t)\|$ these are not vectors

F 20) $\|\mathbf{r}'(t)\|$ is the speed vector not a vector

F 21) $\mathbf{r}''(t)$ may have a component perpendicular to the osculating plane no component outside of osculating plane

F 22) A projectile fired at angle θ with muzzle velocity v_0 has horizontal velocity $v_0 \sin\theta$ cosine

F 23) Range only depends on muzzle velocity launch angle, too

T 24) The Frenét frame is a basis for $\mathbb{R}^3$

T 25) The binormal vector is defined in terms of the normal vector for a curve