FALL - 2024 - CALCULUS 3 - EXAM 1A - Solutions

True or false:

F 1) Every vector in $\mathbb{R}^3$ corresponds to a unique line segment

F 2) A line segment is uniquely determined by its magnitude and direction

T 3) Two vectors can be added using the parallelogram law

F 4) Vectors may be multiplied together in three ways

F 5) There is an angle where the dot product and cross product of two vectors are equal

F 6) The length of vector $\langle a,b,c\rangle$ is $\sqrt{a+b+c}$

T 7) $|(\mathbf{A}\times\mathbf{B})\cdot\mathbf{C}|=|(\mathbf{B}\times\mathbf{C})\cdot\mathbf{A}|$

T 8) There is a vector perpendicular to every vector in $\mathbb{R}^3$

F 9) $\mathbf{A}\times\mathbf{B}$ is perpendicular to $\mathbf{B}$ but not $\mathbf{A}$

T 10) $|\mathbf{A}\times\mathbf{B}|=|\mathbf{A}||\mathbf{B}|\sin\theta$

F 11) The norm of a vector is always greater than zero

F 12) A parallelogram with adjacent sides $\mathbf{A}$ and $\mathbf{B}$ has area $(\mathbf{A}\times\mathbf{B})$

T 13) Perpendicular vectors have zero dot product

T 14) Antiparallel vectors have zero cross product

F 15) A plane is determined by a point on the plane and a vector in the plane

T 16) The equation of a plane thru the point $\mathbf{r}_0$ perpendicular to $\mathbf{v}$ is $(\mathbf{r}-\mathbf{r}_0)\cdot\mathbf{v}=0$

T 17) There are infinitely many planes perpendicular to a fixed direction vector

F 18) The graph of $-\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{25}=1$ consists of one sheet

F 19) The angle between two planes is the same as the angle between lines parallel to each plane

F 20) A force vector $\langle 1,2,1\rangle N$ acting through a displacement vector $\langle 1,-2,3\rangle m$ results in $8J$

F 21) The points $(1,-2,4)$, $(-2,4,-8)$, and $(0,0,0)$ determine a plane

F 22) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C})=(\mathbf{A}\times\mathbf{B})\times\mathbf{C}$

T 23) $c\mathbf{A}\times\mathbf{B}=\mathbf{A}\times c\mathbf{B}$ for constant c

F 24) $\mathbf{A}\times(\mathbf{B}\times\mathbf{A})=\mathbf{0}$ always

T 25) No matter how many forces are acting on a point in $\mathbb{R}^3$, they can be perfectly counterbalanced by one additional force