SPRING 2025 - CALCULUS 2 - TEST #2B - Solutions

1) You have \$1000 invested at 6% per annum continuous interest starting in year 1. At the end of year 3 you withdraw half the money in the account and leave the remainder on deposit. At the beginning of year 4, you get a better rate of 7.5% per annum. How much would you have on deposit at the end of year 10?

Use the Pert formula to find future values. After three years at 6% the \$1000 grows to \$1197.22. Removing half of that leaves \$598.61. Letting that grow for seven years at 7.5% gives \$1011.93.

2) A cup of soup cooled from 90^o C to 60^o C in ten minutes in a room whose temperature was 20^o C. How much longer will it be until the soup is at 35^o C?

Newton's Law of Cooling is $H(t) - H_S = (H_0 - H_S)e^{-kt}$*, where* $H(t)$ *is temperature of an object at time* t *after it has started at temperature* H_0 *to cool off or heat up in an ambient temperature of* H_S*. We need to find* k*, so we plug in our data.* $H(10) = 60$*,* $H_S = 20$*,* $H_0 = 90$*. So* $(60 - 20) = (90 - 20)e^{-10k}$*. Or* $40 = 70e^{-10k}$*. Solving for* k *we have* $\ln(0.5714) = -0.5596 = -k(10)$*, then* $k = 0.05596$*. We want the time* t *to cool from* 60 *to* 35 *with this cooling constant.* $(35 - 20) = (60 - 20)e^{-0.05596t}$*. This gives* $15 = 40e^{-0.05596t}$*, or* $\ln(0.375) = -0.981 = -0.05596t$*. We find* $t = 17.5$ *minutes longer.*

3) Evaluate: $\int_1^4 \frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx$

Let $u = \sqrt{x}$*. Then* $du = \frac{dx}{2\sqrt{x}}$*. So* $\int_1^4 \frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx = \int_1^2 2^u(2du) = \left[\frac{2 \cdot 2^u}{\ln 2}\right]_1^2 = \frac{2}{\ln 2}[2^2 - 2] = \frac{4}{\ln 2}$*.*

4) Plutonium-239 has a half-life of $24{,}360$ days. If 25 [g] of plutonium is released into the atmoshere by a nuclear accident, how many years will it take for 65% of it to decay?

Convert to years: 24360 days = 66.7 years. From the half-life info, $\frac{1}{2} = e^{-\lambda(66.7)}$*, where* λ *is the (positive) decay constant. Solving the exponential equation we get* $-.69315 = -66.7\lambda$*, or* $\lambda = 0.0104$*. Then putting that back into the decay equation, we get* $Q(t) = Q(0)e^{-0.0104t}$*. We want* t *so that* $\frac{Q(t)}{Q(0)} = 0.35$*, so* $0.35 = e^{-0.0104t}$*. Solving this exponential equation we get* $\ln(0.35) = -1.049 = -0.0104t$*, so* $t = 100.9$ *years for 65% to decay or 35% to remain.*