# CALCULUS 2 - SPRING 2025 - EXAM 1B - Solutions

1) Find the volume of the figure obtained by rotating the region between $y = x^2 + 4$ and $y = 2$ around the x-axis from $x = 1$ to $x = 3$.

*Use the washer method. The outside radius is $x^2 + 4$, the inside radius is $2$. So a washer has area $\pi\left[(x^2+4)^2 - 2^2\right] = \pi(x^4 + 8x^2 + 12)$. Then differential volume $dV = \pi(x^4 + 8x^2 + 12)dx$. Integrate this from $x = 1$ to $x = 3$ to get the total volume. So*
$V = \int_1^3 \pi(x^4 + 8x^2 + 12)dx = \pi\left[\frac{x^5}{5} + \frac{8x^3}{3} + 12x\right]_1^3 = \pi[(48.6 + 72 + 36) - (0.2 + 2.67 + 12)] =$
$141.7\pi = 445.3$ or $\frac{2126\pi}{15}$ *cubic units.*

2) Find the surface area of the paraboloid created by revolving $y = \sqrt{x}$ around the x-axis over the interval $[0, 2]$.

Use the surface area formula $S = \int_0^2 2\pi\left(\text{radius}\right)\left(\text{differential line length}\right)$. Since $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$, this works out to $S = 2\pi\int_0^2 \sqrt{x}\left(\sqrt{1 + \frac{1}{4x}}\right)dx$. This integral is the same as $2\pi\int_0^2 \sqrt{x + \frac{x}{4x}}\,dx = 2\pi\int_0^2 \sqrt{x + \frac{1}{4}}\,dx$ Now let $u = x + \frac{1}{4}$ so $du = dx$. In terms of $u$,
$S = 2\pi\int_{0.25}^{2.25} \sqrt{u}\,du = \frac{4\pi}{3}[u^{3/2}]_{0.25}^{2.25} = \frac{4\pi}{3}(3.375 - 0.125) = 13.61$ or $\frac{13\pi}{3}$ square units

3) A dam in a narrow gorge is 80 feet high and has a trapezoidal shape, 100 feet wide at the top and 60 feet at the bottom. Find the total force on the dam if the water behind it is 5 feet from the top. Let the distance from the top of the dam be $x$. So $x$ goes down from $0$ to $80$ feet. Let $w(x)$ be the width of the dam at $x$. Then $w(x) = 100 - 40\left(\frac{x}{80}\right)$ so $w(x) = 100 - \frac{x}{2}$. The differential area of a horizontal strip of the dam $x$ feet from the top is $w(x)dx = \left(100 - \frac{x}{2}\right)dx$. The pressure at $x$ from the top is $62.4(x - 5)$ because the water starts at $x = 5$, so the differential force on that horizontal strip is $62.4(x-5)\left(100 - \frac{x}{2}\right)dx$. The total force on the dam is the sum of these differential forces, so
$F = 62.4\int_5^{80}(x-5)\left(100 - \frac{x}{2}\right)dx = 62.4\left[\int_5^{80}\left(\frac{205x}{2} - \frac{x^2}{2} - 500\right)dx\right]$. This works out to be $1.27237 \times 10^7\ [lbs] = 6361.9\ [tons]$