SPRING 2025 - APPLIED TOPICS - TEST 2A - Solutions

T 1) The Laplace transform is a mapping from time space to frequency space

T 2) Transformable functions must have exponential order

F 3) All continuous or piecewise continuous functions have a Laplace transform - need exponential order

F 4) $\mathcal{L}(f(t)) = \int_0^\infty e^{st} f(t)dt$ - e^{-st}

T 5) $\mathcal{L}(1) = \frac{1}{s}$

F 6) $\mathcal{L}(t^2) = \frac{2!}{s^2}$ - $\frac{2!}{s^3}$

F 7) $\mathcal{L}^{-1}(1) = \delta(0)$ - $\delta(t)$

F 8) $\mathcal{L}(\cos 2t) = \frac{1}{s^2+4}$ - $\frac{s}{s^2+4}$

F 9) $\mathcal{L}(\tan t) = \frac{\mathcal{L}(\sin t)}{\mathcal{L}(\cos t)}$ - just no

F 10) $\mathcal{L}(t-1) = \frac{e^{st}}{s^2}$ - $\frac{e^{-s}}{s^2}$

T 11) $\mathcal{L}(\sin t \cos t) = \frac{1}{s^2+4}$ - $\mathcal{L}(\frac{\sin 2t}{2}) = \frac{2}{2(s^2+4)}$

T 12) $\mathcal{L}((t-3) = \frac{e^{-3s}}{s}$

F 13) $\mathcal{L}(\cosh 2t) = \frac{s}{s^2+4}$ - $\frac{s}{s^2-4}$

F 14) $\mathcal{L}(f^{(3)}(t)) = s^3F(s) - f(0) - sf'(0) - s^2f''(0)$ - powers of s descend

F 15) $\mathcal{L}(tf(t)) = \frac{d}{ds}F(s)$ - $(-1)\frac{d}{ds}F(s)$

T 16) $\lim_{s\to\infty} F(s) = 0$

F 17) $\lim_{t\to 0} f(t) = \lim_{s\to\infty} F(s)$ - $\lim_{s\to\infty} sF(s)$

T 18) Laplace transforms cannot be directly applied to non-linear differential equations - $\mathcal{L}(\sqrt{f(t)}) = ??$

F 19) If $f(t)$ is periodic with period α, then $\mathcal{L}(f(t)) = (1-e^{-s\alpha})\int_0^\alpha e^{-st}f(t)dt$ - $(1-e^{-s\alpha})^{-1}\int_0^\alpha e^{-st}f(t)dt$

T 20) Laplace transforms are linear operators on the space of transformable functions

T 21) Leibnitz's Rule for differentiation under the integral sign requires a partial derivative of the integrand

T 22) Laplace transforms are invertible

F 23) $f(t) = e^{t^2}$ has a Laplace transform - not exponential order

F 24) $\mathcal{L}^{-1}(e^{-as}F(s)) = f(t+a)$ - $f(t-a)$

F 25) $\mathcal{L}^{-1}\left(\frac{(n+1)!}{s^n}\right) = nt^{n-1}$ - $n(n+1)t^{n-1}$