ALGEBRA 2 - SPRING 2025 - TEST 3B

References OK. Be careful not to assume properties that are not explicit.

1) Divide $x+2$ into $2x^7-3x^6+5x^5-7x^4+4x^2+x-\beta$ and determine β so that it divides evenly

2) Show that $8x^3+14x^2-91x+23$ does not factor over $\mathbb{Z}$.

3) Show that in the ring $\mathbb{Z}\left[\sqrt{n^2+1}\right]$, there are at least four units...$n \geq 1$

4) Write down a polynomial over $\mathbb{Z}$ that has degree 7, every power of x present, has content 5, and is irreducible over $\mathbb{Q}$. Then show why.

5) Show that the ideal $\langle 19, 43\rangle$ is not maximal in $\mathbb{Z}$?