ALGEBRA 2 - SPRING 2025 - TEST 3A - Solutions

T 1) If D is a domain, so is $D[x]$ proved in class

T 2) If F is a field, $F[x]$ is a unique factorization domain recent theorem

T 3) A non-zero, non-unit element in a domain D that is prime is also irreducible over D proved in class

T 4) An element that is irreducible over a domain D may be reducible over domain $D' \subset D$ $2x+4$ irred over $\mathbb{Q}$ but reducible over $\mathbb{Z}$

T 5) If $f(x) \in \mathbb{Z}[x]$ is reducible over $\mathbb{Q}$ is reducible over $\mathbb{Z}$ big theorem

T 6) The norm of $2-\sqrt{-5}$ in the quadratic number field $\mathbb{Z}\left[\sqrt{-5}\right]$ is 9 $2^2-(1)(-5)$

T 7) Every euclidean domain is a unique factorization domain $ED \Rightarrow PID \Rightarrow UFD$

F 8) The homomorphism $\phi : \mathbb{Z} \to \mathbb{Z}_n$ given by $x \mapsto x \bmod n$ has $\ker\phi = \{0\}$ any multiple of n

F 9) The First Isomorphism Theorem for rings says that if $\phi : R \to S$ is a homomorphism and A is an ideal of R, then R/A is isomorphic to S. $\operatorname{Im}\phi$

T 10) There is a field with exactly 120 invertible elements. $\mathbb{Z}_{11^2}$

T 11) Formal quotients of polynomials $\dfrac{f(x)}{g(x)}$, where $f(x), g(x) \in \mathbb{Z}[x]$ and $g(x) \neq 0$ are a field field of fractions

F 12) $\mathbb{Z}[x]$ is a euclidean domain not a PID so can't be an ED

T 13) If $f(x)$ is reducible over $\mathbb{Q}$ then it factors over $\mathbb{Z}$ see (5)

T 14) The euclidean algorithm applies to polynomials over $\mathbb{Q}$ field

F 15) $f(x) = x^3-4x^2+3x-2$ is reducible over $\mathbb{Z}_5$ use $\bmod p$ theorem

T 16) $f(x) = x^5-7x^3+3x^2-4x+1$ has a zero in $\mathbb{R}$ graph crosses x-axis for all odd degree polys

F 17) If F is a field, distinct polynomials in $F[x]$ represent distinct functions from $F[x]$ to F - look @ x^3 vs x^2 over $\mathbb{Z}_2$

F 18) Every field has a proper subfield $\mathbb{Q}$ does not

F 19) The Eisenstein criterion only applies to monic polynomials any leading coefficient

T 20) A primitive polynomial times a polynomial with content C results in a polynomial with content C use Gauss' lemma

T 21) Monic polynomials are always primitive $\gcd$ has to be 1

F 22) Primitive polynomials are always monic just as long as $\gcd$ is 1

T 23) If F is a field, a quadratic over F times a cubic over F always gives a quintic (degree five) over F leading coefficients not zero divisors

T 24) A subfield can be treated as a vector space over the containing field going to be very important next chapter

F 25) $\langle x^5+1\rangle$ is a maximal ideal in $\mathbb{Q}[x]$ reducible over $\mathbb{Z}$, hence $\mathbb{Q}$...$(x+1)(x^4-x^3+x^2-x+1) = x^5+1$