ABSTRACT ALGEBRA - SPRING 2025 - EXAM 2B (no 2A)

All honorable references permitted. Due in class Mar 11 or earlier as pdf.

1) Show that ring homomorphisms map nilpotent elements into nilpotent elements. Recall $a \in R$, a ring, is nilpotent of index $n \in \mathbb{N}$ if $a^n = 0$ and n is the smallest integer for which this is true.

2) Let E and F be fields. If the rings $E[x]$ and $F[x]$ are ring-isomorphic, show that E and F are also ring-isomorphic.

3) Let F be a field and R be a ring. If $\phi : F \to R$ is a homomorphism, show that $\phi(F)$ is either the nullring or $\phi(F)$ is a field.

4) Show that if ϕ and ψ are endomorphisms of ring R, then so is $\phi \circ \psi$

5) Let $I = \{f(x) \in \mathbb{Z}[x]$: sum of coefficients of $f(x)$ is $0\}$. Show that $\mathbb{Z}[x]/I \cong \mathbb{Z}$