ABSTRACT ALGEBRA - SPRING 2025 - TEST 1B

All honorable references permitted. God and the KGB are watching. Due Feb 4. All problems equal value.

1) Decide if $4x^2 + 6x + 3$ is a unit in $\mathbb{Z}_8[x]$ and, if so, find its inverse.

2) Find all maximal ideals in $\mathbb{R} \oplus \mathbb{R} \oplus \mathbb{R}$.

3) Show that $\{0, 2, 4, 6, 8, 10, 12\}$ is a field modulo 14. What is the identity?

4) Suppose that $u, v,$ and $u + v$ are units in a commutative ring. Show $u^{-1} + v^{-1}$ is also a unit.

5) Let R be a commutative unital ring. Let I be a proper ideal of R with the property that every element in $R - I$ is a unit. Show I is a unique maximal ideal of R.