SPRING 2025 - ABSTRACT ALGEBRA 2 - TEST 1A - Solutions

True or false:

false 1) A ring with zero divisors is an integral domain   no zero divisors plus commutative and 1

false 2) A commutative ring with no zero divisors is an integral domain   need 1

true 3) $\mathbb{Q}[x]$ is an integral domain   since $\mathbb{Q}$ is a field, hence domain

true 4) Cancellation holds in a PID   cancellation holds in domains of any type

true 5) $\Bbbk[x]$ is an integral domain if $\Bbbk$ is a field   field implies domain

true 6) A field must have at least one nonzero element   1 always present and $0 \neq 1$

true 7) If u is a unit, so is $-u^2$ multiply by $-u^{-2}$

false 8) In a principal ideal domain every element is a multiple of a single element   in the ideal, not the whole ring

false 9) The center of a ring is an integral domain   no guarantee of 1 (think $2\mathbb{Z}$)

false 10) $\mathbb{Z}_8$ is an integral domain $2 \cdot 4 = 0$

true 11) If D is an integral domain, so is $D[x]$ theorem

false 12) If $a|bc$ in a ring then $a|b$ or $a|c$ $6|3 \cdot 4$

false 13) If I is a subring of a commutative unital ring R, then R/I is a factor ring   need ideal

false 14) If I is an ideal of a ring R, then R/I is an integral domain   factor ring only

false 15) If I is a maximal ideal of a ring R, then R/I is a field   need 1

false 16) If I is a prime ideal of a commutative ring R, then R/I is an integral domain need 1

false 17) If I is a prime ideal of a commutative ring R and $ab \in I$, then $a \in I$ maybe b

false 18) If I is an ideal of a commutative unital ring R, then R/I is a field   ideal must be maximal

true 19) If I is a maximal ideal of an integral domain R, then R/I is a field   commutativity and 1 present

false 20) Two maximal ideals of the same ring are disjoint   share $\{0\}$

false 21) If I is an ideal of a commutative unital ring R, then R/I is a field   see 18

false 22) $12\mathbb{Z}$ has four zero divisors   no but $\mathbb{Z}_{12}$ does...2,6,3,4

true 23) All maximal ideals are prime   theorem

true 24) There are infinitely many ideals of $\mathbb{Z}$ $p\mathbb{Z}$, p prime

true 25) All ideals of $\mathbb{Z}$ are principal