CALCULUS 2 - FALL 26 - EXAM 1B - Solutions

1) Find the volume of the figure obtained by rotating $y = \sqrt[3]{x}$ about the x-axis from 0 to 8

This is a disk method problem. $V = \pi \int_0^8 (\sqrt[3]{x})^2 dx = \pi \int_0^8 x^{2/3} dx = \pi[\frac{3}{5}x^{5/3}]_0^8 = \frac{3\pi}{5}(32) = \frac{96\pi}{5}$

2) Find the surface area (neglect the bottom) of a cone with height 2 and slant height 3

Orient the cone with apex at origin and cone axis coinciding with the x axis. Base radius of cone (by Pythagorean theorem) is $\sqrt{3^2 - 2^2} = \sqrt{5}$, so side extends from origin to point $\left(2, \sqrt{5}\right)$. Equation of side is then $y = \frac{\sqrt{5}}{2}x$. Plug into surface area of revolution formula.
Area
$= 2\pi \int_0^2 \left(\frac{\sqrt{5}}{2}x\right)\left(\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right)dx = \pi\sqrt{5} \int_0^2 x\left(\sqrt{1 + \frac{5}{4}}\right)dx = \pi\sqrt{5} \int_0^2 x\left(\frac{3}{2}\right)dx = \frac{3\pi\sqrt{5}}{2}\left[\frac{x^2}{2}\right]_0^2 = 3\pi$

3) An industrial hopper is full of sand weighing 125 lbs/cf. The hopper itself is an inverted square based pyramid with side 6 ft and depth 4 ft. The hopper is emptied by vacuuming the sand over the top. How much work would be done?
A section thru a square pyramid perpendicular to its axis is also going to be a square. We define x to be the depth into the hopper from the top, so x will go from 0 to 4. We need the cross sectional area at arbitrary depth x. Viewed from the side of the hopper we see a triangle with side 6 ft at the top tapering to zero at the bottom, and an altitude of 4 ft. Let $w(x)$ be the width of the triangle at depth x By similar triangles we have $\frac{w(x)}{4-x} = \frac{6}{4}$ (this is just the common ratio of base to altitude). Solving for $w(x)$ we get $w(x) = 6 - \frac{3}{2}x$. Then the area of this section is $\left(6 - \frac{3}{2}x\right)^2$ and the weight of the differential layer of sand at depth x is $125\left(6 - \frac{3}{2}x\right)^2 dx$. This amount of sand requires the vacuum to lift it x feet to be removed from the hopper. So the differential work is $dW(x) = 125x\left(6 - \frac{3}{2}x\right)^2 dx$. Finally, integrating this from 0 to 4 gives us total work $= \int_0^4 125x\left(6 - \frac{3}{2}x\right)^2 dx = 6000$ ft-lbs.

4) A cylindrical propane tank is 15 feet long and has diameter 5 feet. The ends are closed with hemispherical caps. The gas pressure is 500 psi. What is the total force on one of the hemispherical caps? (Don't overthink this)

The total force on either of the endcaps is the same as the force that would be developed if the cylinder were capped with a flat disk. Such a disk would have area $\pi(2.5)^2$, so the total force would be $\left(500\ \frac{lbs}{in^2}\right)\left(144\ \frac{in^2}{ft2}\right)\left(6.25\pi\ ft^2\right) = 1,413,720$ lbs. or about 707 tons force.