FALL 2019 - ABSTRACT ALGEBRA - FINAL A - PRACTICE

Unless otherwise noted, $\overline{G}$, $G \geq H$ are groups (not necessarily abelian), $\phi : G \to \overline{G}$ is a homomorphism, $a, b, g, h \in G, H$, p is a prime, $n \in \mathbb{N}$

True or false:

1) Every coset is also a subgroup
2) If $gH = Hg$ for all $g \in G$ then G is a normal group
3) If p divides $|G|$ then G has a subgroup of order p
4) $[G : H]$ divides the order of H
5) If $H \geq K$, then $|K|$ divides $|G|$
6) If $N \lhd G$, then $HN \lhd G$
7) If $g \in aH \cap bH$, then $aH = bH$
8) $(2,2) \in \mathbb{Z}_2 \oplus \mathbb{Z}_2$
9) $|(2,3)|$ in $\mathbb{Z}_4 \oplus \mathbb{Z}_4$ is 6
10) $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ has three subgroups of order 2
11) If $N \lhd G$ then $|G| = |G/N| \cdot |N|$
12) If $G/Z(G)$ is abelian then G is cyclic
13) Every group of order p^2 is abelian
14) Every group whose order is the product of two primes is abelian
15) $\ker\phi \lhd G$
16) $im\phi \lhd \overline{G}$
17) A homomorphism between any two groups always has an inverse
18) If $a, b \in \ker\phi$ then $ab = ba$
19) If G is cyclic, then $\phi(G)$ is cyclic
20) If G is abelian, then $\phi(G)$ is abelian
21) $\phi(\ker\phi) \leq \overline{G}$ is always abelian
22) $a\ker\phi = b\ker\phi$ implies $\phi(a) = \phi(b)$
23) $G/\ker\phi \cong \overline{G}$
24) There is only one abelian group of order ten
25) There is only one abelian group of order nine