FALL 2022 - ABSTRACT ALGEBRA 1 - TEST 1A - Solutions

Unless otherwise noted, G,H are groups, $g,h \in G,H$, p is a prime, $n \in \mathbb{N}$

No references

True or false:

T 1) Every dihedral group D_n has at least five abelian subgroups - trivial, n flips of order 2, rotation of order n where $n \geq 3$

F 2) The Klein group is cyclic - no generator

T 3) If an element in G has order n, then G has a subgroup of order n - Theorem 3.4

T 4) An infinite group may have a finite subgroup - $\langle 0 \rangle < \mathbb{Z}$

T 5) Right cancellation is valid for group operations - right multiply by inverse

T 6) A monoid is a group without an inverse for every element - definition

F 7) The dihedral group D_{2n} has $2n$ elements - $4n$

F 8) $U(p)$ has p elements - $\{1,2,\cdots p-1\}$

F 9) The group product $g_1 \cdots g_k$ never depends on the order of factors - depends on order, not grouping

F 10) All abelian groups are cyclic - other way around, V_4 is abelian but not cyclic

T 11) A group may have an infinite number of subgroups - $\{n\mathbb{Z} \leq \mathbb{Z} : n \in \mathbb{N}\}$

F 12) The odd integers form a group under addition - no identity

T 13) A group of units is always abelian - modular multiplication is commutative

F 14) The center of a group is never trivial - D_3 is centerless (by def'n only identity is in center)

T 15) The union of two subgroups of G can be a proper subgroup of G - if one subgroup is already contained in the other

F 16) $\mathbb{Z}_{10} \leq \mathbb{Z}_5$ - as elements, yes $\{0,\pm 10,\cdots\} \subset \{\{0,\pm 5,\pm 10,\cdots\}$ but the group operations are different

T 17) A surjective map between two finite sets of equal cardinality is always injective - "to fill the seats, no one can sit on someone's lap"

F 18) If $g \in G$, then $C(g) \subset Z(G)$ - If G is nonabelian, then $Z(G) \subsetneq G$, but $C(e) = G$, always

F 19) Every group can be partitioned into its subgroups - partition blocks are pairwise disjoint, but any collection of subgroups would share the identity

F 20) Any abelian subgroup of G must be contained in $Z(G)$ - rotation group of D_3 not contained in center $\langle R_0 \rangle$

F 21) If G is nonabelian, then it is never true for two distinct elements $a,b \in G$ that $ab = ba$ - $eg = ge\ \forall g \in G$

F 23) The union of two proper subgroups of G can equal G - for union to be subgroup, one must be in other, but then largest would have to be G, so improper

F 24) If $G = \langle g^9 \rangle$, then $\langle g^4 \rangle \subsetneq \langle g^2 \rangle$ - $\{g^4,g^8,g^3,g^7,g^2,g^6,g^1,g^5,g^0\} = \{g^2,g^4,g^6,g^8,g^1,g^3,g^5,g^7,g^0\}$

F 25) $3\mathbb{Z} \subset 6\mathbb{Z}$ - other way