FALL 2025 - ABSTRACT ALGEBRA 1 - TEST 3A

Unless otherwise noted, G, H are groups, N is normal subgroup, e is an identity, $g, h \in G, H$, p, q are primes, $n \in \mathbb{N}$, ϕ, ψ are homomorphisms or isomorphisms

No references

1) Define homomorphism of G to H

2) Define direct product of G and H

True or false:

3) In any group the center is normal
4) In any group a subgroup is normal
5) In any abelian group a subgroup is normal
6) In any group the trivial subgroup is normal
7) If $H \leq G$, then G/H is a factor group
8) If $N \leq G$, then G/N is a factor group
9) Th elements of a factor group are cosets
10) If $gN \in G/N$, then $Ng \in G/N$
11) If $gHg^{-1} \subset H$ for $H \leq G$ and any $g \in G$, then $H \triangleleft G$
12) If G/Z is cyclic, so is G
13) If $G/\langle e \rangle$ is cyclic, G is abelian
14) $U(n) \oplus U(m)$ has mn elements
15) $U(p) \oplus U(q)$ has pq elements
16) $\mathbb{Z}_5 \oplus \mathbb{Z}_7$ has 35 elements
17) The order of $(1,1) \in \mathbb{Z}_5 \oplus \mathbb{Z}_3$ is 15
18) There is always a homomorphism between two groups
19) There is always an isomorphism between two groups of the same order
20) The kernel of $\phi : G \to H$ is $\phi(e)$
21) The kernel of a homomorphism is always normal
22) If the kernel of a surjective homomorphism is $\{e\}$, it is a bijection
23) If the kernel of a surjective homomorphism is $\{e\}$, it is an isomorphism
24) The kernel of an isomorphism can be nontrivial
25) If $G = H_1 \oplus H_2$ then $H_1 \cap H_2 = \varnothing$