FALL 2026 - ABSTRACT ALGEBRA 1 - TEST 1A - Solutions

Unless otherwise noted, G, H are groups, $g, h \in G, H$, p is a prime, $n \in \mathbb{N}$, D_{2n} is the dihedral group of the regular n-gon

F 1) D_8 has exactly four subgroups of order 2 five

T 2) The Klein group is abelian

T 3) The order of a subgroup divides the order of the group Lagrange's Theorem

F 4) An infinite group cannot have a finite subgroup $\{1, -1\} \leq \mathbb{Q}^*$

F 5) Right cancellation in a group can be valid even though left cancellation is not always both

T 6) Every inverse in a group is bilateral

F 7) The cyclic group of order 7 has three distinct subgroups only itself and the trivial group

T 8) $U(p)$ has $p-1$ elements everything less and relatively prime

F 9) $GL(3, \mathbb{Z})$ is commutative matrix groups in general are not

T 10) All cyclic groups are abelian powers of single element commute

F 11) $\phi(n)$ is the set of positive integers less than n, relatively prime to n the number of, not the set of

F 12) The even integers form a group under multiplication addition, not multiplication

T 13) A group of units is always abelian modular multiplication is commutative

T 14) The center of a group is never the null set identity is always there

T 15) The union of two subgroups of G can be a proper subgroup of G but one contains the other

T 16) $4\mathbb{Z} \subset 2\mathbb{Z}$ any multiple of 4 is a multiple of 2

T 17) $2\mathbb{Z} \cap 3\mathbb{Z} = 6\mathbb{Z}$ $6n = 2 \cdot 3 \cdot n$

T 18) If $a \in G$ does not belong to $C(g)$ for some $g \in G$, then $a \notin Z(G)$ a doesn't commute with every element

T 19) $\{R_0, R_{180}\} \leq D_8$ the fifth subgroup of order 2

F 20) Any abelian subgroup of G must be contained in $Z(G)$ may not commute with rest of G

F 21) If G is nonabelian, then it is never true for two distinct elements $a, b \in G$ that $ab = ba$ look at rotations in D_{2n}

T 22) If $Z(G) = G$, then G is abelian by definition

T 23) The Cayley table for V_4 is the same as that for $U(8)$ if you re-label $e = 1, a = 3, b = 5, c = 7$ only two groups order 4

T 24) $U(5)$ is generated by 2 $2^4, 2^1, 2^3, 2^2 = 1, 2, 3, 4$ mod 5

T 25) There is a group which has an infinite chain of subgroups, each one containing the next properly: $G \supset H \supset K \supset \cdots$ $2\mathbb{Z} \geq 4\mathbb{Z} \geq 8\mathbb{Z} \geq \cdots 2^n\mathbb{Z} \geq \cdots$