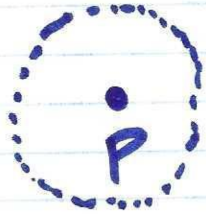
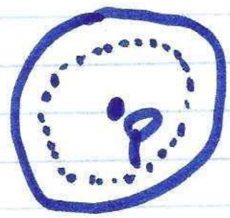
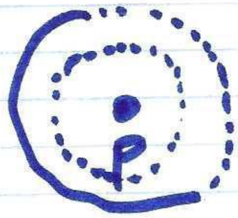
① 9/5



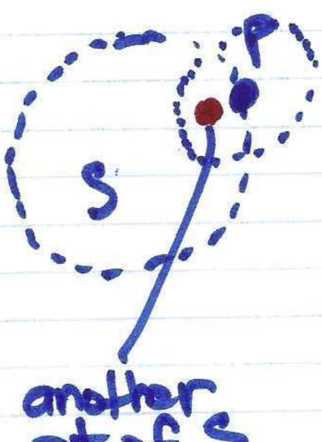

point in an open nbhd



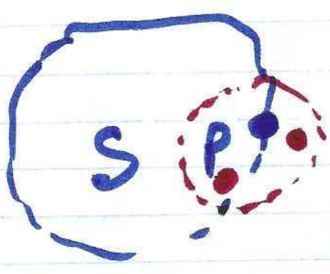

point in closed nbhd




point in arbitrary nbhd




p is a limit point of S because all open sets containing p (i.e. open nbhds) intersect S in a point other than p




p is a boundary point of S if every nbhd of p intersects both S and $S^c$

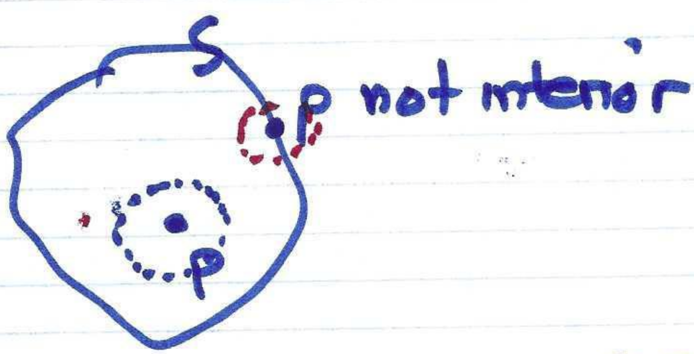
limit point of $S$

accumulation pt.
cluster pt
derived pt

set of all limit points (derived pts) is called the derived set of $S$, denoted $S'$

$\text{cl}\, S$ = closure of $S$ = $S \cup S'$

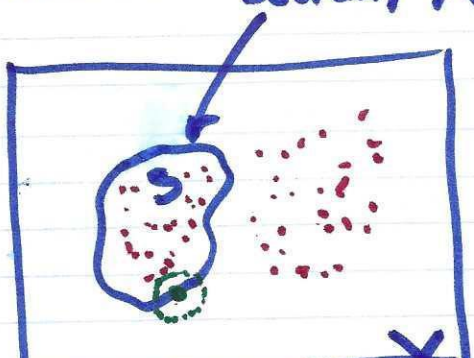
interior point of $S$ is a point contained in an open set itself contained in $S$




int $S$ = {interior pts of $S$}

ext $S$ = int $S^c$

boundary $X \setminus$ int $S \setminus$ ext $S$

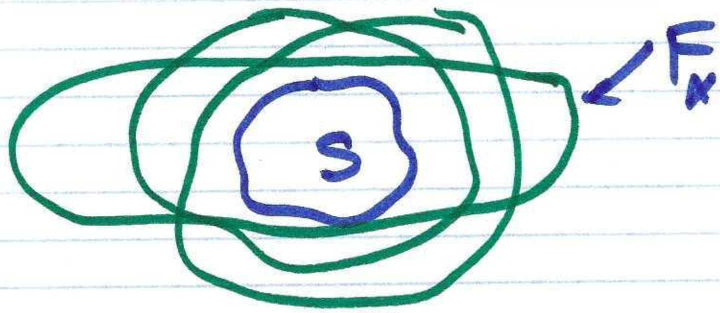



this boundary

F <u>fermé</u> 

$\operatorname{int} S = \bigcup O_\alpha$ where $O_\alpha \subset S$

$\operatorname{cl} S = \bigcap F_\alpha$ where $F_\alpha \supset S$ ; $F$ closed

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closure point (belongs to $\operatorname{cl} S$)

adherent point (adherence)

---

A topological space is a pair $\langle X, \tau )$ where $X$ is any ($\neq \emptyset$) set and $\tau \subset 2^X$. $\tau$ is <u>the</u> topology

(i) $\tau = \{ O_\alpha \}_{\alpha \in \Lambda}$ $\quad O_\alpha$ are called <u>open</u>

(ii) $O_1 \cap O_2 \in \tau$

(iii) $\bigcup_{\alpha \in \Lambda} O_\alpha \in \tau$

$X$ & $\phi$ are always in $\tau$.

$$X = \bigcap_{\alpha \in \phi} \mathcal{O}_\alpha$$

$$\phi = \bigcup_{\alpha \in \phi} \mathcal{O}_\alpha$$

---

Given $\{a,b\}$, What are possible topologies

$$\tau_{dis} = \{\{a\}, \{b\}\} = \underline{\{\phi, \{a\}, \{b\}, \{a,b\}\}}$$

$$\tau_{ind} = \{\phi, \overset{X}{\{a,b\}}\}$$

$$\tau\ (\text{Sierpiński Topology}) = \{\phi, \{a,b\}, \{a\}\}$$

---

$$\tau = \left\{\{[n, n+1, n+2, \dots]\}\right\}_{n \in \mathbb{N}}$$

is a topology

Usual topology on $\mathbb{R}, \mathbb{R}^2, \ldots$

A <u>base</u> for $\langle \mathbb{R}, \mathcal{U} \rangle$ is set of open intervals $\{(a,b) : a, b \in \mathbb{R}, a<b\}$

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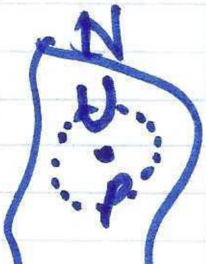
Hausdorff's Axioms - intuitionistic version

There is an object defined for every point $p$ in a ground space $X$ called the neighborhood system (filter). It is denoted by $\mathcal{N}_p$

① $\mathcal{N}_p \neq \emptyset$ & $p \in N \in \mathcal{N}_p \quad \forall N$

② $N_1, N_2 \in \mathcal{N}_p \Rightarrow N_1 \cap N_2 \in \mathcal{N}_p$




③ Given $N \in \mathcal{N}_p \Rightarrow$ if $M \supseteq N$, $M \in \mathcal{N}_p$

④ Each $N \in \mathcal{N}_p$ is a superset of some $U \in \mathcal{N}_p$ where $U$ is a nbhd of <u>each</u> of its own pts.