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Cantor's Thm/ A countable union of countable sets is countable

Pf: (induction)

Base Case: Given A & B with cardinality $\aleph_0$, denote their elements as a_i, b_j respectively. Now $C = A \cup B$ can be written $\{a_1, b_1, a_2, b_2, \cdots\}$. Reindex as $\{c_1, c_2, c_3, \cdots\}$

$$\left.\begin{array}{ll} a_1 \to c_1 & b_1 \to c_2 \\ a_2 \to c_3 & b_2 \to c_4 \\ a_3 \to c_5 & b_3 \to c_6 \end{array}\right\} \text{so mapping}$$

$a_n \to c_{2n-1}$

$b_m \to c_{2m}$

It is clear that every element of A or B is accounted for as some C. C is also countable as the elements are indexable. So base claim is established.

②

Ind. Hyp. $\bigcup_{i=1}^{n} A_i$ is countable if A_i is countable.

Now show $\bigcup_{i=1}^{n+1} A_i$ is countable.

By assumption $\bigcup_{i=1}^{n} A_i$ is countable.

Also A_{n+1} is countable (given). Apply argument from base case to conclude

$$\bigcup_{i=1}^{n} A_i \cup A_{n+1} = \bigcup_{i=1}^{n+1} A_i \text{ is countable.} \blacksquare$$

An equiv. rel'n on set S is:

R, a subset of cartesian product $S \times S$ such that

$s \sim s$ ① $(s,s) \in R \quad \forall s \in S$ (reflexive)

$s \sim t \Leftrightarrow t \sim s$ ② $(s,t) \in R$ iff $(t,s) \in R$ symmetric

$s \sim t,\ t \sim w \Rightarrow s \sim w$ ③ (s,t) & $(t,w) \in R$, then $(s,w) \in R$ (transitivity)

③ $d : M \times M \to \mathbb{R}_{0+}$

Metric Space M contains points like p, q, r etc. & a distance function d.

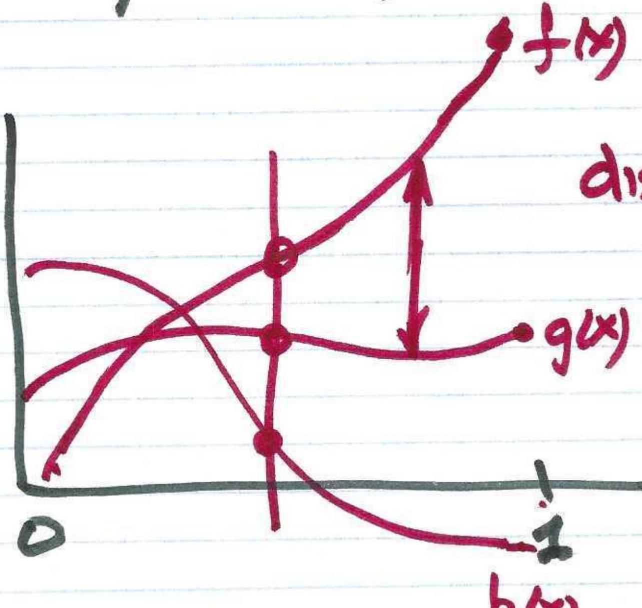
M1 : $d(p,q) = 0$ iff $p = q$ (faithfulness)

M2 : $d(p,q) = d(q,p)$ (symmetry)

M3 : $d(p,q) + d(q,r) \geq d(p,r)$ ($\Delta \neq$)

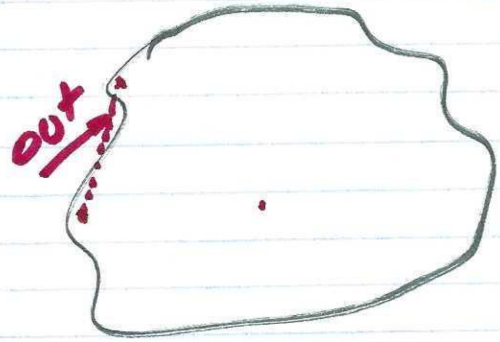
Classic : $d : \mathbb{R}^k \times \mathbb{R}^k \to R_{0+}$

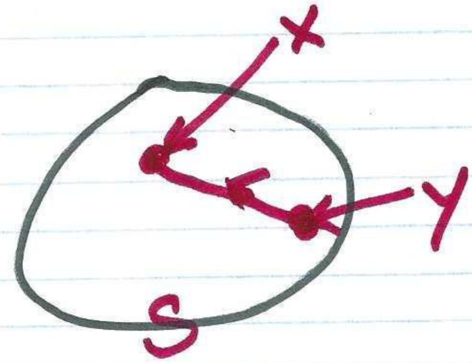
$d(x,y) = |x - y|$ $\quad \nu : \mathbb{R}^k \to \mathbb{R}_{0+}$



$\text{dist}(f,g) = \sup_{x \in [0,1]} |f - g|$

$x(f)$



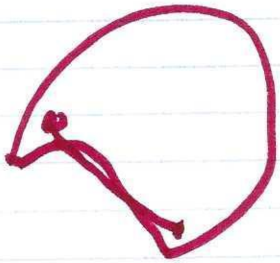


convex body in $\mathbb{R}^k$

S is convex iff

$\underline{\lambda x + (1-\lambda) y} \in S$

$\lambda \in [0,1]$



convex in $\mathbb{R}^2$

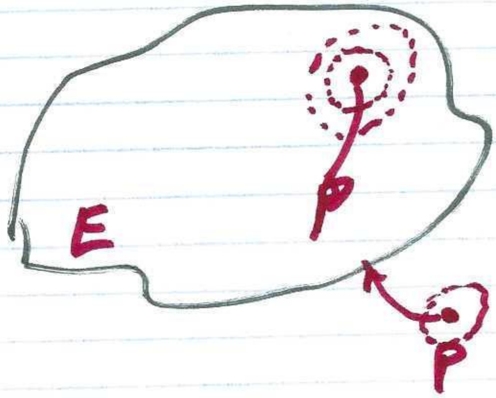
In a metric space M,

i) a neighborhood of a point p is the set $N_p(r)$ with radius r centered @ p where $q \in N_p(r)$ iff $d(p,q) < r$.

r is the "radius of nbhd"

2) A point p is a limit point of set E whenever every nbhd of p contains a point of E other than p.

"deleted nbhd"



(a, b)

3) If $p \in E$ but p not limit pt of E, p is an isolated point.

4) Set is closed if it contains all its limit pts.

5) A general nbhd of point x is any set that contains x along with an open set containing x inside the arbitrary

$[a, b)$ is nbhd of any $\in (a, b)$