We say a space (metric) is separable iff there exists a countable dense subset.

If $\overline{\mathbb{Q}} = \mathbb{R}$ we say rationals dense in reals.

#22 Show $\mathbb{R}^k$ is separable

$$\mathbb{R}^k = \prod_{i=1}^{k} \mathbb{R}_i \,, \quad \mathbb{R}_i = \mathbb{R}$$

Convergence

In a metric space, sequence $p_1, p_2, \cdots, p_n \cdots$

converges to point p iff following happens

$B_p(\varepsilon)$
$B(p, \varepsilon)$ For any metric ball $B_p(\varepsilon)$ of radius $\varepsilon > 0$ containing p, we may find an $N \in \mathbb{N}$ such that for $n > N$ it is true that $p_n \in B_p(\varepsilon)$.

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in $\mathbb{R}^1$ We say $p_1, p_2, \cdots p_n \cdots$ converges to p in $\mathbb{R}$ if given $\varepsilon > 0$ $\exists N(\varepsilon)$ such that $n > N(\varepsilon) \Rightarrow p_n \in (p - \varepsilon, p + \varepsilon)$.

Filters

A non-void collection of non-void sets in some ground space X is called a filter ($\mathcal{F}$) iff:

(i) $A, B \in \mathcal{F} \Rightarrow A \cap B \in \mathcal{F}$

(ii) $A \in \mathcal{F}$ and $B \supseteq A$, then $B \in \mathcal{F}$

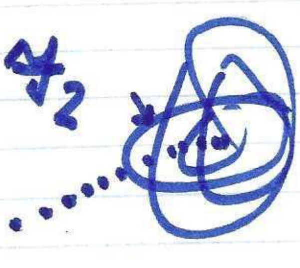
Given a sequence $a_1, a_2, \cdots a_n \cdots$

We say the elementary filter of the sequence is the collection of all co-finite sectional sets of elements.

Elementary filter is an example of a section filter.

$\mathcal{F}_1$ refines $\mathcal{F}_2$ if $\mathcal{F}_1 \supset \mathcal{F}_2$

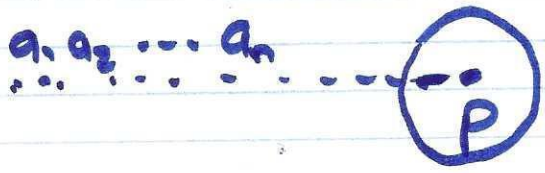
$\mathcal{F}_2$ coarsens $\mathcal{F}_1$



The sequence $a_1, a_2 \cdots$ converges to $p \in X$ whenever the elementary filter of sequence refines the nbhd system of p.

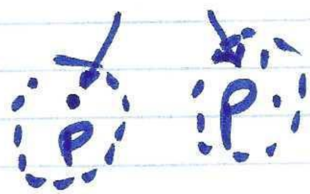
$\to p$

p.18 (c) $S_n = 1 + \frac{(-1)^n}{n}$

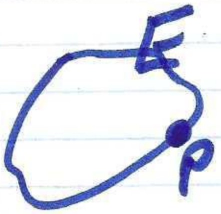


3.2 Th^{m}

(b)



Ex: $\frac{1}{1} + 2 + \frac{1}{3} + 4 + \cdots$



$\langle p_n \rangle \to p$

$\lim_{n\to\infty} e^{p_n} = e^{p}$

$p_n > 0$

$p > 0$

$\lim_{n\to\infty} \ln(p_n) = \ln p$

4

$$\mathbb{R}^{\mathbb{N}} = \{(\alpha_1, \alpha_2, \cdots \alpha_k \cdots), \alpha_k \in \mathbb{R}\}$$

$$d(x,y) = \sqrt{(\alpha_1'-\alpha_1)^2 + (\alpha_2'-\alpha_2)^2 + \cdots (\alpha_k'-\alpha_k)^2}$$

$$x = (\alpha_1, \alpha_2 \cdots \alpha_k)$$

$$y = (\alpha_1', \alpha_2' \cdots \alpha_k')$$

$$\text{in } \mathbb{R}^{\mathbb{N}} = \sqrt{\sum_{i=1}^{\infty} |\alpha_i - \alpha_i'|^2}$$

Hilbert sequence space

l_2