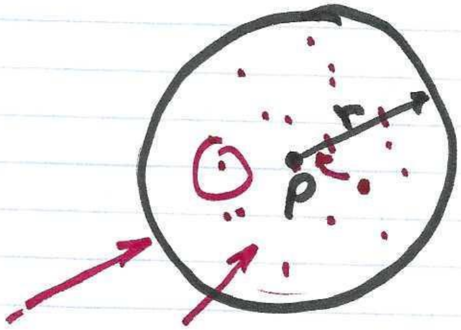
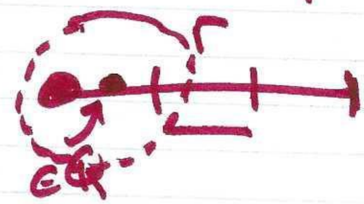
9/12

Hmwk #2



(r,s) $r,s \in \mathbb{Q}$

$\rho \in \mathbb{Q}$



$B_\rho(p)$ rational

Compactness

1) Borel-Lebesgue arbitrary
2) Countable compactness
3) Bolzano-Weierstrass (limit point)
4) Sequential compactness

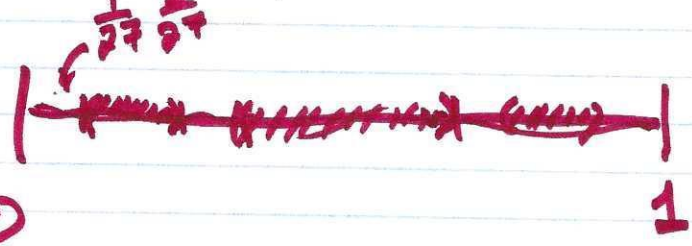
$[n, \infty)$ $n \in \mathbb{N}$

$$\bigcap_{n \in \mathbb{N}} [n, \infty] = \emptyset$$

$$\bigcap_{n \in \mathbb{N}} \left(-\frac{1}{n}, n\right) = \{0\}$$

$|\mathbb{R}| > \aleph_0$ diagonalization

Cantor set



$I_1 = [0;1]$

$I_2 = [0, 1/3] \cup [2/3, 1]$

$I_3 = [0, 1/9] \cup [\frac{2}{9}, \frac{1}{3}] \cup [\frac{2}{3}, \frac{7}{9}] \cup [\frac{8}{9}, 1]$

$\downarrow$

$K = \bigcap_{n=1}^{\infty} I_n$ K is compact

$0 + \frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \cdots \frac{2^{n-1}}{3^n}$

$I_1 \quad I_2 \quad I_3$

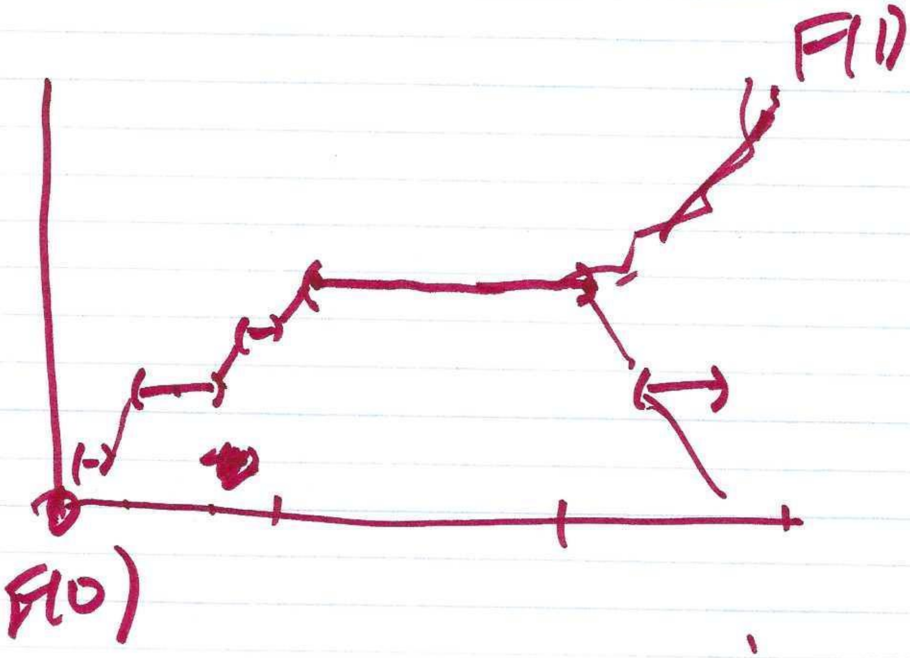
remove: $\sum_{n=1}^{\infty} \frac{2^{n-1}}{3^n} = \frac{1}{2} \sum_{n=1} \left(\frac{2}{3}\right)^n = \frac{1}{2} \cdot \frac{2/3}{1 - 2/3} = 1$

③

0.0- - .

0.100-→0

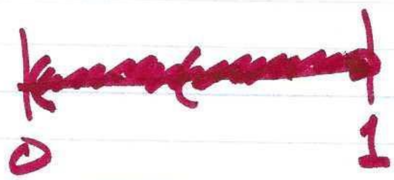
0.200--→0



$$F(1) - F(0) = 1 * \int_0^1 K(x)\,dx$$

1. Given S, show $\emptyset \subset S$.
2. done
3. done
4. done
5. Bounded set of reals w/ 3 limit points

$$\bigcup_{n=1}^{\infty} \left(\frac{1}{n}, 1\right] =$$



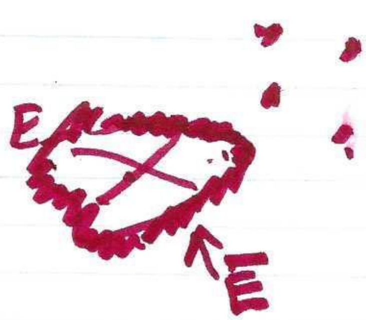
$(0, 1]$

$$\left\{\frac{1}{n}\right\}_{n \in \mathbb{N}}$$

$1, \frac{1}{2}, \frac{1}{3} \dots$ 0

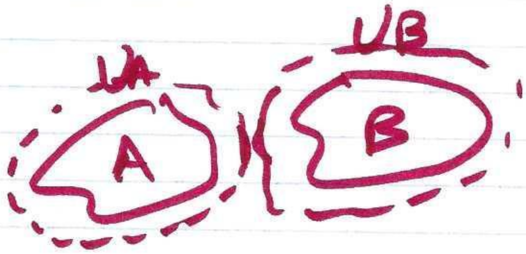
$$\left\{\frac{1}{n}\right\}_{n \in \mathbb{N}}, \cup \left\{1 + \frac{1}{n}\right\}_{n \in \mathbb{N}} \cup \left\{2 + \frac{1}{n}\right\}_{n \in \mathbb{N}}$$

0   1   2

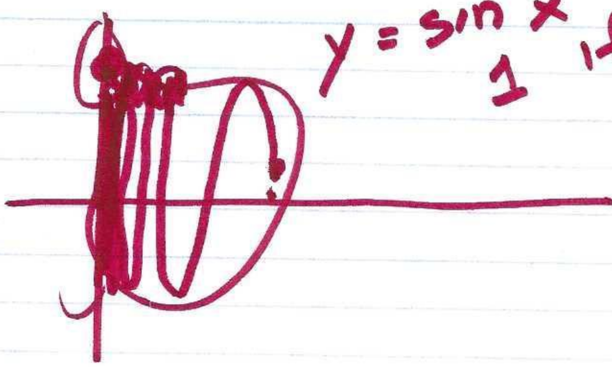


Two sets A & B are separated if $\exists\, U_A$ & U_B, open such that

$U_A \cap U_B = \emptyset$ & $A \subset U_A$, $B \subset U_B$.



$$y = \begin{cases} \sin\frac{1}{x} & x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$



$(0,1)$, $(1,2)$ are disconnected