8/29

Ex. of induction proof:

Prove $\sum_{k=1}^{n} k = S(n) = \frac{n(n+1)}{2}$

P(a) Base Case - true $\frac{(1)(2)}{2} = 1$ ✓

$P(k) \Rightarrow P(k+1)$

ind step

Induction Hypothesis:

True for $n = m$ i.e. $S(m)$ true

Want to show true for $n = m+1$

$S(m) = \frac{m(m+1)}{2}$ ← "assumed"

$S(m+1) = S(m) + (m+1)$

$\frac{m(m+1)}{2} + (m+1) =$

$\frac{m(m+1)}{2} + \frac{2m+2}{2} = \frac{m^2+m+2m+2}{2}$

$= \frac{m^2+3m+2}{2} = \frac{(m+1)(m+2)}{2} = S(m+1)$

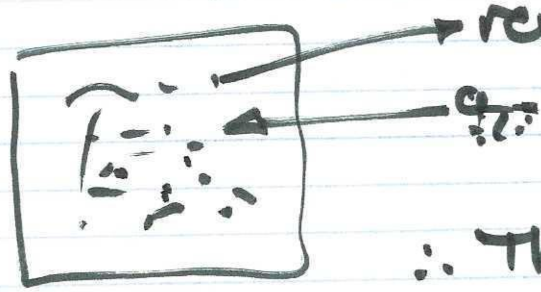
$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

Bovine Monochromaticty Lemma.

All cows are same color.

Base Case: Bossy, a cow, is some color α.

Assume any herd of n cows is monochrome.



remove 1 cow Elsie

restored

∴ The herd with

all $n+1$ cows is monochrome

n cows

∎

$X^Y \ni f : Y \to X$

Given set A, card $A <$ card $\mathcal{P}(A) = 2^A$

Two sets have same cardinality iff $\exists \phi$

where ϕ is a bijection between them.

Given sets A & B

if $\exists f : A \to B$ and

f is injective, then card $A \leq$ card B

f is surjective, then card $A \geq$ card B

k-set K n-set N

n^k possible maps

What if $n = 2$

$K \to \{0,1\}$

subsets of K is 2^K

④ x $\{x\}$

Cantor's Thm/ $\underline{\text{card } A} < \text{card } 2^A$

Assume FSOC that $\text{card } A = \text{card } 2^A$

So $\exists \phi : A \to 2^A \ni \phi$ is bijective.

Define $x \in A$ is "good" if $x \in \phi(x)$

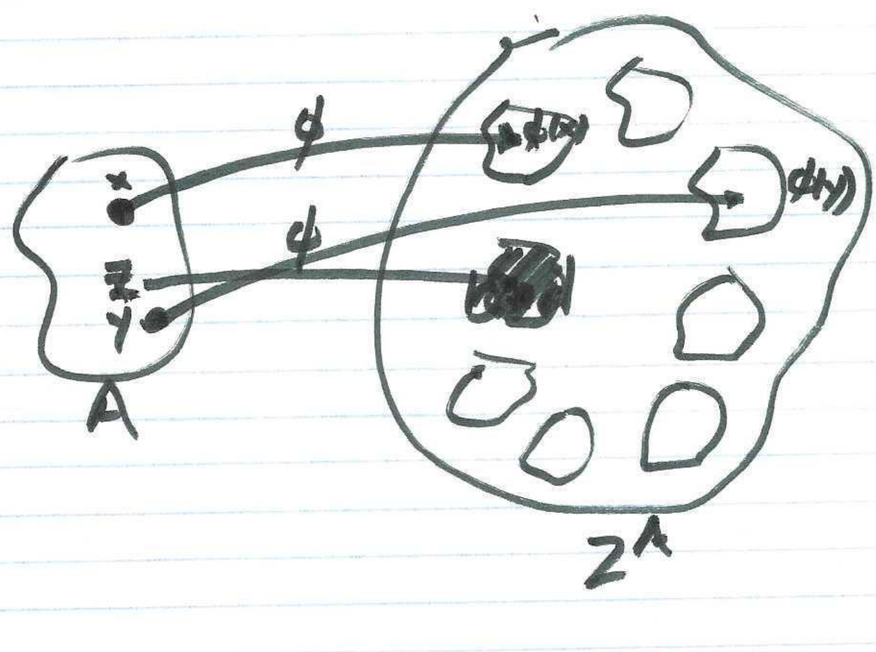
$x \in A$ is "bad" if $x \notin \phi(x)$

Consider B = set of all bad elements

$$B = \{x \in A : x \notin \phi(x)\}$$

$\exists x_B \in A \ni \phi(x_B)$ is subset of all bad elements.

$$\left.\begin{array}{l} x_B \text{ is good} \Rightarrow x_B \text{ is bad} \\ x_B \text{ is bad} \Rightarrow x_B \text{ is good} \end{array}\right\} \Rightarrow\Leftarrow$$



aleph scale

$\aleph_0$ countable

$2^{\aleph_0} = \aleph_1$ accept continuum hypothesis

$2^{\aleph_1} = \aleph_2$

↓

$\beth_0 = \aleph_0$

beth

$\beth_1 = 2^{\beth_0} \overset{?}{\neq} \aleph_1$

$\beth_n = 2^{\beth_{n-1}}$

Algebraic number is the root of some polynomial w/ integer coeff.

$5, \frac{7}{6}, \ldots \sqrt{2}, \sqrt{19}, \ldots \pi$

$x^2 - 2 = 0$ NO!

Thm $\text{card}(A) = \aleph_0$

Given $p(x) = a_n x^n + \cdots a_1 x + a_0$

define $h(p) = \sum |a_i| + \partial p$

$h(x^2+3) = 2+1+3 = 6$

$h(2x^2-5) = 2+2+5 = 9$

ht	# poly's
0	0
1	± 1
2	$x, -x, \pm 2$
3	$\pm x \pm 1 \; \pm 3$
⋮	

p_0

p_1, p_2

p_3, p_4, p_5, p_6

$\downarrow$