

G

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$$\textcircled{1} \quad r \in \mathbb{Q} \quad x \notin \mathbb{Q}$$

a) where is $r+x$ Let $r = \frac{\sqrt{3}}{c}$

$$\frac{\sqrt{3}}{c} + x = \frac{r}{c} = x = \frac{r}{c} - \frac{\sqrt{3}}{c} = \frac{nu - mv}{\sqrt{3}}$$

b) where $r \cdot x$

$$\frac{\sqrt{3}}{c} \cdot x = \frac{r}{c} \Rightarrow x = \frac{r}{\sqrt{3}} \in \mathbb{Q} \Rightarrow \text{No}$$

$$\textcircled{2} \quad \left[(\sqrt{2})^{\frac{1}{\sqrt{2}}} \right]^{\sqrt{2}} = (\sqrt{2})^{\frac{1}{\sqrt{2}} \cdot \sqrt{2}} = \sqrt{2} \quad a^{k \cdot \frac{1}{n}} = \sqrt[n]{a^k}$$

$$\textcircled{3} \quad |x| + |y| \geq |x+y|$$

$$|x| + |z| \geq |x+z| \quad \text{Let } z = y-x$$

$$|x| + |y-x| \geq |y|$$

$$||x|-|y|| \geq |x-y|$$

(2)

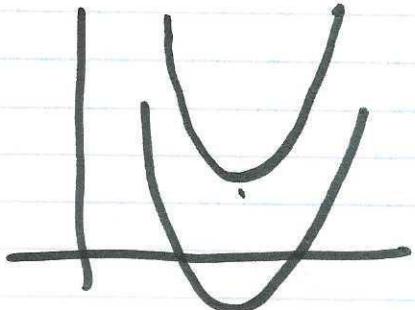
$$x = \langle x_1, x_2, x_3, x_4 \rangle \quad y = \langle y_1, y_2, y_3, y_4 \rangle$$

$$x \cdot y = \frac{\sum_{i=1}^4 x_i y_i}{\sqrt{|x| |y|}} \leq |x| |y|$$

$$-1 \leq \frac{x \cdot y}{|x| |y|} \leq 1$$

$$\text{Show } \left(\sum_{i=1}^n a_i b_i \right)^2 \leq \sum_{i=1}^n a_i^2 \cdot \sum_{i=1}^n b_i^2$$

$$\sum_{i=1}^n |\bar{a}_i x - b_i|^2 \geq 0$$



$$\sum_{i=1}^n (a_i x - b_i)^2 \geq 0$$

$$\sum_{i=1}^n (a_i^2 x^2 - 2a_i b_i x + b_i^2) \geq 0$$

$$\left(\sum_{i=1}^n a_i^2 \right) x^2 - 2 \left[\sum_{i=1}^n a_i b_i \right] x + \left(\sum_{i=1}^n b_i^2 \right) \geq 0$$

A B

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$$\sqrt{\Delta} = \sqrt{4\left(\sum_{i=1}^n a_i b_i\right)^2 - 4\left(\sum_{i=1}^n a_i^2\right) \cdot \left(\sum_{i=1}^n b_i^2\right)}$$

$$\sqrt{\frac{\Delta}{2}} = \sqrt{\sum_{i=1}^n a_i b_i^2} - \sum a_i^2 \cdot \sum b_i^2$$

$$\Rightarrow \left(\sum_{i=1}^n a_i b_i \right)^2 - \left(\sum_{i=1}^n a_i^2 \right) \left(\sum_{i=1}^n b_i^2 \right) \leq 0$$

$$\Rightarrow \left(\sum_{i=1}^n a_i b_i \right)^2 \leq \left(\sum_{i=1}^n a_i^2 \right) \left(\sum_{i=1}^n b_i^2 \right)$$

$$A \cdot B \leq |A| \cdot |B|$$

Cauchy - Schwarz - Bonyakowski

A hand-drawn timeline on lined paper. A horizontal black line represents time, starting at the left edge and ending at the right edge of the page. Above the line, there are vertical tick marks. The first two tick marks are blue and slanted upwards to the left. The third tick mark is red and slanted downwards to the left. The fourth tick mark is black and vertical. The fifth tick mark is red and slanted downwards to the left. The sixth tick mark is black and vertical. The seventh tick mark is red and slanted downwards to the left. The eighth tick mark is black and vertical. The ninth tick mark is red and slanted downwards to the left. The tenth tick mark is red and vertical. The eleventh tick mark is red and slanted upwards to the left. The twelfth tick mark is red and vertical.

Why doesn't C have an order like R?

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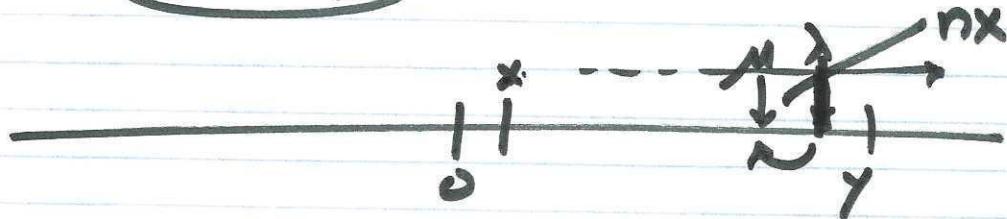
$$\begin{array}{l} \cancel{i^2 < 0} \Rightarrow -i < 0 \quad -\cancel{i} > 0 \\ i^2 > 0 \Rightarrow -i > 0 \end{array}$$

④

\mathbb{R} has the archimedean property :

Given $x > 0$ $y > 0 \exists n \in \mathbb{N} \text{ s.t. }$

$$nx > y$$



Pf: Suppose FSCC no such n exists.

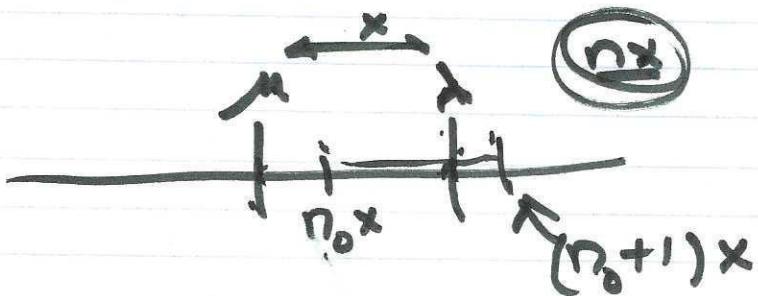
If so y is upper bound $\{nx : n \in \mathbb{N}\}$

By completeness axiom (i.e. l.u.b. exists)
we have l.u.b. $\lambda > nx \forall n$

$$\mu = \underline{(n-1)x}$$

Some element $n_0 x > \mu$

Consider $(n_0 + 1)x > \lambda \rightarrow$



(5)

$$\mathbb{R}^n = R \times R \times \cdots \times R$$

$$d(x, y) = \sqrt{\sum_{i=1}^n (x_i - y_i)^2} \leftarrow \text{euclidean "metric"}$$

$$\langle \mathbb{R}^n, d \rangle = E^n \text{ euclidean } n\text{-space}$$

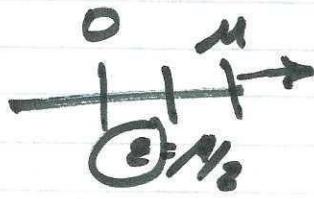
$$\boxed{|x+y|^2} = (x+y)^2 = \underline{x^2 + 2xy + y^2}$$

$$2xy \leq 2|x||y|$$

$$\leq x^2 + 2|x||y| + y^2 =$$

$$|x|^2 + 2|x||y| + |y|^2 = \boxed{(|x| + |y|)^2}$$

$$\boxed{|x+y|^2 \leq (|x| + |y|)^2}$$



If $x < \varepsilon$ for all $\varepsilon > 0$

What is x ? If not, say $x > m > 0$

then choose $\varepsilon = m/2 \Rightarrow *$

(6)

Ground space (i.e. like \mathbb{R} or \mathbb{Q} or \mathbb{C})

Define family of sets that are "stable" under arbitrary unions and finite intersections.

So, if X is ground space

$\mathcal{P}(X) = 2^X$ is set of all subsets of X .

i) for unions if Ω_α belongs to family

then $\bigcup_{\alpha \in A} \Omega_\alpha$ also belongs to family

ii) for $\Omega_1 \cap \Omega_2 \in \text{family}$

$$\left(\frac{1}{n}, \frac{1}{n}\right) \quad \cancel{\overbrace{\hspace{1cm}}^0}$$

$\langle \{\Omega_\alpha\}_{\alpha \in A}, X \rangle$ is a topology