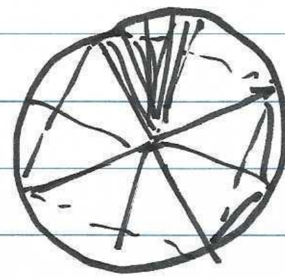
1 Integration - oldest

Eudoxian "integral"



↓

Fermat

↓

Age of Intuition | Newton/Leibnitz $\int \frac{d}{dx} f'(x)$

↓

Euler Analysin Infinitorum (1748)

$$\frac{1}{z-1} - \frac{1}{z-1} = 0 \qquad z \neq 1$$

$$\frac{1}{z-1} - \frac{1}{z\left(1-\frac{1}{z}\right)} \qquad \frac{1}{1-z} = 1+z+z^2+\cdots z^n$$

↓

$$-\frac{1}{1-z} - \frac{1}{z\left(1-\frac{1}{z}\right)} \quad \Rightarrow \quad \frac{1}{1-\frac{1}{z}} = 1 + \frac{1}{z} + \frac{1}{z^2}\cdots$$

$$\times\frac{1}{z}$$

$$\frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3}\cdots$$

↓

$$-\left(1+z+z^2+\cdots z^n\cdots\right) - \left(\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+\cdots\frac{1}{z^m}+\cdots\right)$$

$$-\left[\cdots z^n\cdots+z^2+z+1+\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+\cdots\frac{1}{z^m}+\cdots\right] = 0$$

Age of Rigor - <u>prove</u> in some fashion the claims of earlier mathematicians

A. Cauchy (1821) <u>Cours d'Analyse</u>

<u>Lagrange</u>

<u>uniform convergence</u> Cauchy misunderstood this

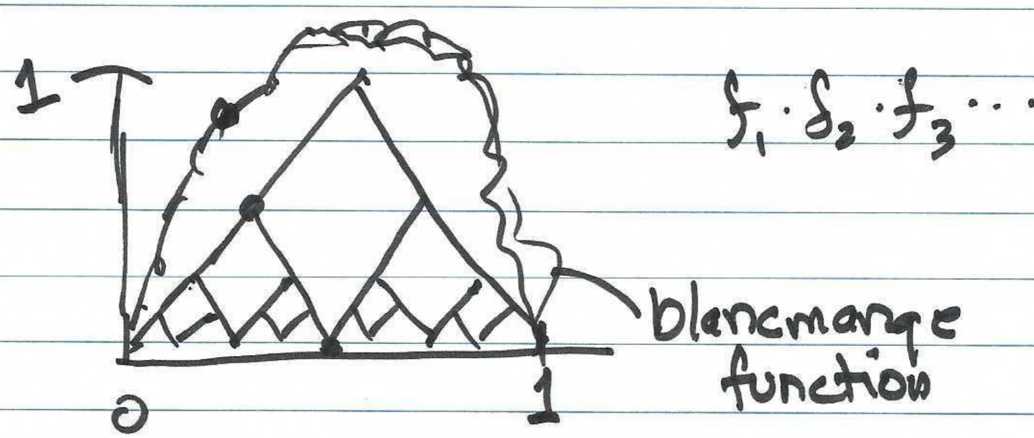
Karl Weierstrass solved the uniformity

↓

Darboux
Riemann

↓

1880's Monsters appeared



Formalist School · advent of axiom systems
(ZFC) Zermelo-Fraenkel-Choice

1903 - Russell's Paradox

Cantor c. 1875 developed "Naïve Set Theory".

Consider the set of all sets.

Now look @ set of all sets "that are not members of themselves".

Let M be this set. So two possibilities

$$\begin{cases} M \in M \Rightarrow M \notin M & \underline{\text{by def'n}} \\ M \notin M \Rightarrow M \in M \end{cases}$$

. ZFC → enjoin any use of all sets

+ "this sentence is false"

$\alpha \Rightarrow \alpha \cup \{\alpha\}$ Burali-Forti Paradox

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→ $\underline{\emptyset \text{ exists}}$ $\{\emptyset\}$ $\{\emptyset, \{\emptyset\}\}$ . . .

$$\begin{array}{ccc} \emptyset & \{\emptyset\} & \{\emptyset, \{\emptyset\}\} \\ \downarrow & \downarrow & \downarrow \\ 0 & 1 & 2 \end{array}$$

Neumann-Bernays-Gödel Axiom System (classes)