$\{a_n\}_N \quad \left|\sum_{n=1}^{\infty} a_n\right| \le \sum_{n=1}^{\infty} |a_n| < \infty$

$a_n \in \mathbb{R}$

Power Series:

$$\sum_{n=1}^{\infty} c_n z^n$$

$$\limsup_{n\to\infty} \sqrt[n]{|a_n| z^n} < 1 \Rightarrow \text{conv}$$

$$= |z| \overbrace{\left(\limsup_{n\to\infty} \sqrt[n]{|a_n|}\right)}^{\beta} < 1$$

$$\text{so } |z| < \frac{1}{\beta}$$

$$\text{or } |z|\beta < 1 \quad \text{let } \beta = \frac{1}{R}$$

$$\text{so } \frac{|z|}{R} < 1$$

$$\text{so } a_n = \frac{1}{z^n}$$

2

(i) $\sum_n n^n z^n$ $\quad \limsup_{n\to\infty} \sqrt[n]{n^n} = +\infty$

$$R = \frac{1}{+\infty} = 0$$

(ii) $\sum_n \frac{z^n}{n!} = e^z$ $\quad \limsup_{n\to\infty} \sqrt[n]{\frac{1}{n!}} = 0$

$R = +\infty$ $\qquad n! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$ (Stirling)

$$\sqrt[n]{\frac{1}{n!}} = \frac{e}{n} \cdot \frac{1}{\sqrt{2\pi n}}$$

(iii) $\sum_n z^n$ $\qquad R = 1$

(iv) $\sum_n \frac{z^n}{n}$ $\quad \limsup_{n\to\infty} \sqrt[n]{\frac{1}{n}} =$

$R = 1$ $\qquad \lim \frac{1}{\sqrt[n]{n}} = 1$

(v) $\sum_n \frac{z^n}{n^2}$ $\frac{1}{n^2} < \frac{1}{n}$

$$\limsup_{n\to\infty} \sqrt[n]{\frac{1}{n^2}} = \lim \frac{1}{\sqrt[n]{n^2}} =$$

$R = 1$ $\frac{1}{\sqrt[n]{n}\cdot\sqrt[n]{n}}$

(vi) $\sum_n \frac{z^n}{n^k}$ $k = 1, 2, \cdots$

Dirichlet's Th^m

$\sum_n a_n b_n$ and $a_n \downarrow 0$ &

$\sum_{k=1}^{n} b_k < C < \infty$

$\sum_{n=1}^{K} \sin n$

$\sum_n \frac{\sin n}{n} = ?$

4

Two series $\sum a_n x^n \sum b_n x^n$

$$\left(\sum_{n=0}^{\infty} a_n x^n\right) \cdot \left(\sum_{n=0}^{\infty} b_n x^n\right)$$

Define $c_n = a_0 b_n + a_1 b_{n-1} + a_2 b_{n-2} + \cdots +$

$a_{n-1} b_1 + a_n b_0$

$$c_n = \sum_{k=0}^{n} a_k b_{n-k}$$ ← convolution of a_k, b_k

Cauchy Product

$$\int f(x)\, g(t-x)$$

$$\mathcal{L}(f) = \int_0^\infty f(t) e^{-st}\, dt$$

freq time

Merten's Thm