Thm's & Facts

$\mathbb{R}$ is ordered field $\mathbb{R}$ is complete
$\mathbb{C}$ " not $\mathbb{C}$ " "
$\mathbb{Q}$ " not

Cauchy-Schwarz Inequality ✓

$\Delta\neq$ $|x|+|y| \geq |x+y|$

$\Delta\neq'$ $|x-y| \geq \big||x|-|y|\big|$

If $0 \le x < \varepsilon$ for all $\varepsilon > 0$, $x = 0$

Set DeM: $\left(\bigcup_\alpha S_\alpha\right)^c = \bigcap_\alpha S_\alpha^c$

$\left(\bigcap_\alpha S_\alpha\right)^c = \bigcup_\alpha S_\alpha^c$

Metric def^{n} & axioms

Metric balls are convex

Nbhd of limit point p has ∞-ly many points
(so p is an ω-limit point)

Condensation point -
every nbhd has uncountably many pts

$\aleph_0$ card of $\mathbb{Q}, \mathbb{N}$

$\aleph_1$ " " $\mathbb{R}, \mathbb{C}$ (assuming CH)

$2^{\aleph_0} > \aleph_0$

Countable union of countable sets is countable.

E is open if E^c is closed

$\bigcup_{\text{any}}$ open is open

$\bigcup_{\text{finite}}$ closed is closed

$\bigcap_{\text{any}}$ closed is closed

$\bigcap_{\text{finite}}$ open is open

full space X is clopen

y limit pt of E & E closed $\Rightarrow y \in E$

metric thms

K compact in $X \Rightarrow K$ closed

F closed $\subset K$ compact $\Rightarrow F$ compact

Heine-Borel Every k-cell is compact

Nested (closed) Interval Th^{m}

Compact Sets are B-W compact

Sequences: All subsequences of a convergent seq. are conv. (to seq. limit) if non-stat.

All sequences in $\mathbb{R}^k$ are cauchy.

Compact M.S. are sequentially compact

diam E

Root/Ratio Tests

Def^{n}'s.

archimedean
order relation
equiv relation
sup
inf
boundedness
countability / uncountability
cardinality
k-tuple
rational number
lub property
metric
metric space
limit point
nbhd
isolated point
closed set
closure
interior point
complement
perfect set
dense
topology
filter
filterbase
compact set
Borel-Lebesgue condition

co-ordinates
cover
open cover
subcover
relative compactness
k-cell
Cantor set
connected set
separated sets
seq. converge
seq. diverge
finite range
max vs sup
min vs inf
subsequence
cauchy seq.
completeness
lim sup
lim inf
extended $\mathbb{R}$
convexity

1. field is archimedean if ___
Given $x \in \mathbb{R}$, $\exists n \ni n \geq x$

$\dot{\ddot{\,}}$ = "and"

2. order relation on set X

Set of pairs (x_1, x_2) w/ flwg:

$\leq$ $\qquad R \subseteq X \times X$

$\underline{(x_1, x_1)}$
1. $x_1 \leq x_1$
2. $x_1 \leq x_2$ $\dot{\ddot{\,}}$ $x_2 \leq x_1 \Rightarrow x_2 = x_1$
3. $x_1 \leq x_2$ $\dot{\ddot{\,}}$ $x_2 \leq x_3 \Rightarrow x_1 \leq x_3$

3. equivalence relation $\quad S \subseteq X \times X$

1. $x = x$
2. $x = y \Rightarrow y = x$
3. $x = y$ $\dot{\ddot{\,}}$ $y = z \Rightarrow x = z$

4. supremum least upper bound

5. infimum greatest lower bound

6. $B \subset \mathbb{R}$ is bounded if

$\exists M \ni (-M, M) \supset B$

7. Countable - 1:1 correspondance with set of integers

(8) Uncountable - can't be counted by $\mathbb{N}$

(9) Cardinality - # elements in set

(10) k-tuple from X $(x_1, x_2, \dots, x_k)$

(11) rational number $\{\frac{m}{n} : m, n \in \mathbb{N}, n \neq 0\}$

(12) metric space $\langle X, d \rangle$

① $d(x,y) = 0$ iff $x = y$

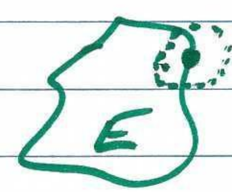
② $d(x,y) = d(y,x)$

$\Delta\neq$ ③ $d(x,y) \leq d(x,z) + d(z,y)$

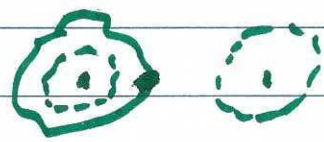
(13) $\Delta\neq$ $|x| + |y| \geq |x+y|$

$(\Delta\neq)'$ $\big||x| - |y|\big| \leq |x-y|$

(14) limit point of set $E \subset X$



(15)



(16) topology on X, τ

τ is a family of <u>open</u> sets

stable under arb. $\bigcup$ & finite $\bigcap$

Set rel'ns in topology

$\bigcup_{\infty}$ open is open

$\bigcap_{\infty}$ closed is closed

C_X

$\bigcup_{n}$ closed is closed

$\bigcap_{n}$ open is open

17 isolated pt. of $E \subset X$

$\cancel{\bar{E}}$

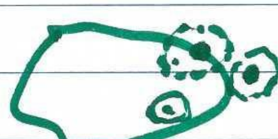
18 perfect – no isolated pts.

E^c

19 closed set has <u>all</u> limit points cl E

20 closure of set

21 interior point int E E°

22  boundary set

23 If A is dense in $B \Leftrightarrow \underline{\mathrm{cl}A \supset B}$

24 Separable B is separable if

$\mathrm{cl}A \supset B$ and $\mathrm{card}(A) = \aleph_0$

$\emptyset$

25 Filter is a family of sets $(\neq \emptyset)$

stable under finite $\cap$ & supersets

26 Compact set - every open cover has a finite subcover

27 Connected Set - no disconnection

28 Disconnection of set E is

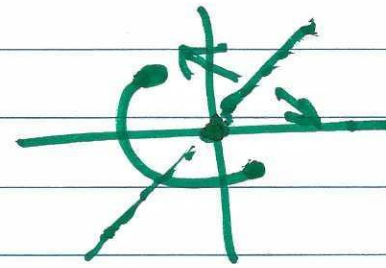
a pair of set S_1 & S_2 such that

$E \cap S_1 \neq \emptyset,\ E \cap S_2 = \emptyset$

$E \subset S_1 \cup S_2$

$S_1 \cap \bar{S}_2 = \bar{S}_1 \cap S_2 = \emptyset$

$E_1 \cup E_2 = E$



29 Convergence of sequences

$\langle a_i \rangle_{i \in \mathbb{N}}$

Fix $\varepsilon > 0$ If $\exists N \ni n > N$ $\underbrace{|a_n - L|}$ $< \varepsilon$.

$\mathbb{R}^{\mathbb{N}}$ metric $\to d(a_n, L)$

30 Subsequence $a_1\ a_2\ a_3\ a_4\ a_5\ a_6 \ldots$

 2 3

seq : $\mathbb{N} \to A$

$\mathbb{N} \to \mathbb{N}$ monotone