$\lim_{n\to\infty} \sqrt[n]{n} = 1$ $\quad$ $x_n = 1 - \sqrt[n]{n}$

$1 + nx \le (1+x)^n \ldots$

$$(1+x)^n = 1 + \sum_{k=1}^{\infty} \frac{x^k}{k!} n^{\underline{k}} \qquad |x| < 1$$

$$n^{\underline{k}} = n(n-1)\cdots\overset{(n-k+1)}{(n-(k-1))} \qquad P(n,k) \quad {}_nP_k$$

$$(1+x)^n \ge 1 + \frac{x^1}{1!}(n) = 1 + nx + \_\_\_\_$$

$$\ge 1 + \frac{x^2}{2} n^2$$

Maclaurin Series

$$f(x) = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \cdots$$

$$(1+x)^n \qquad 1 + nx + \frac{(n)(n-1)x^2}{2} + \cdots$$

$$n(1+x)^{n-1}$$

$$n(n-1)(1+x)^{n-2}$$

$$m(v) = \frac{m_0}{\sqrt{1 - v^2/c^2}} = m_0\left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

$$E = mc^2 = c^2 m_0\left[1 + \left(-\frac{1}{2}\right)\left(-\frac{v^2}{c^2}\right) + \left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{v^2}{2c^2}\right)^2 + \cdots \right. \quad O\left(\frac{v^6}{c^6}\right)$$

$$= m_0c^2 + \frac{v^2}{2\cancel{c^2}} \cdot \cancel{c^2} m_0 + \frac{3}{8}\frac{v^4}{c^4} + O\left(\frac{v^6}{c^6}\right)$$

$$= m_0c^2 + \frac{m_0v^2}{2} + \frac{3}{8} m_0 \frac{v^4}{c^2} + \cdots$$

$$a_n = \sqrt{\pi a_{n-1}}$$

$$a_1 = \sqrt{\pi} \quad \pi^{1/2}$$

$$a_2 = \sqrt{\pi \cdot \sqrt{\pi}} = \sqrt{\pi^{3/2}} = \pi^{3/4} \qquad a_n \uparrow$$

$$a_3 = \sqrt{\pi \cdot \pi^{3/4}} = \pi^{7/8}$$

$$\pi^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}} \rightarrow$$

$$S = a_0 + \cancel{a_0 r} + \cancel{a_0 r^2} + \cancel{\cdots}$$

$$rS = \cancel{r a_0} + \cancel{a_0 r^2} + \cancel{a_0 r^3} + \cancel{\cdots}$$

$$S - rS = a_0 \quad \text{or} \quad S = \frac{a_0}{1-r} \qquad \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$$

$$\sum_{n=1}^{\infty} a_n$$

$$S_1 = a_1$$

$$S_2 = a_1 + a_2$$

$$S_3 = a_1 + a_2 + a_3$$

$$\vdots$$

$$\sum_{n=1}^{\infty} \frac{1}{4n^2-1} = \frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{2n-1} - \frac{1}{2n+1}$$

$n=1$ $\quad 1 - \frac{1}{3}$

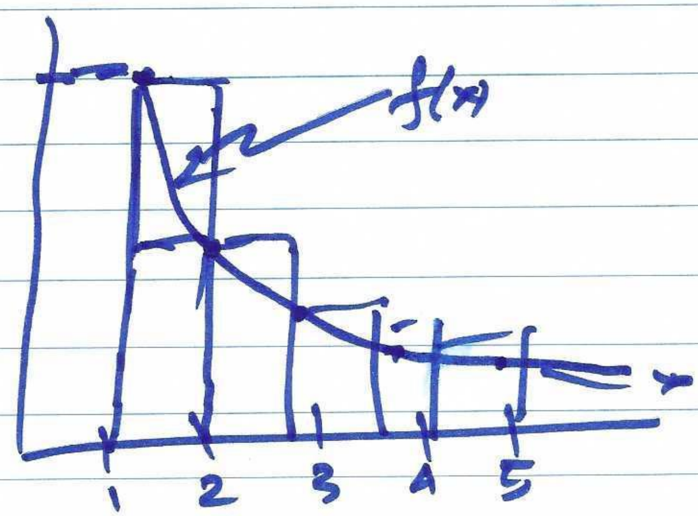
$n=2$ $\quad \frac{1}{3} - \frac{1}{5}$

$n=3$ $\quad \frac{1}{5} - \frac{1}{7}$

$f(x)$ and $a_n = f(n)$

$\int_1^{\infty} f(x)\,dx$ & $\sum_{n=1}^{\infty} a_n$ conv/div together

$\frac{1}{x}$ $\quad \frac{1}{n} = a_n$ $\qquad \int_1^{\infty} \frac{1}{x}\,dx = [\ln x]_1^{\infty} = \infty$



$$\underbrace{\frac{1}{2}}_{=\frac{1}{2}} + \underbrace{\frac{1}{3} + \frac{1}{4}}_{>\frac{1}{2}} + \underbrace{\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}}_{>\frac{1}{2}} + \underbrace{\frac{1}{9} + \frac{1}{10} + \frac{1}{11} + \cdots}_{>\frac{1}{2} \rightarrow}$$

Pringsheim's Criterion

if

$na_n \to 0$

$a_n \to 0$ nec <u>not suff</u>