LOGIC - LECTURE 4

Validity Testing

Let's dive in with an example, then analyze what we have done and why.

Let p and q stand for sentence variables. We could give them meaning - semantic content - but we want full generality.

Here is an argument:

Premises:

$$1)\ \neg(p \cdot q) \supset \neg(p \vee q)$$

$$2)\ \neg(p \cdot \neg q)$$

$/\therefore$ Conclusion: $\neg(q \cdot \neg p)$

Question: Is this argument form valid?

Why is an argument ever invalid? Simple - it has true premises but a false conclusion. This suggests a brute force examination of all possibilities to test for validity.

We make a table with 4 rows:

		premise 1					premise 2			conclusion		
p	q	$\neg$	$(p \cdot q)$	$\supset$	$\neg$	$(p \vee q)$	$\neg$	$(p \cdot$	$\neg q)$	$\neg$	$(q \cdot$	$\neg p)$
T	T	F	T	T	F	T	T	F	F	T	F	F
T	F	T	F	F	F	T	F	T	T	T	F	F
F	T	T	F	F	F	T	T	F	F	F	T	T
F	F	T	F	T	T	F	T	F	T	T	F	T

I have given each of two sentences a possible true (T) or false (F) value.

A truth table with n sentences will need 2^n rows to cover all combinations.

The premises are written linearly at the top, and under each major operator (in red) the possible truth values for each premise, given the assumed truth values of p & q are listed. Likewise for the conclusion. Now focus on the red columns. The only time the conclusion can be false is row 3, and in that row, premise 1 is false. There are no cases (rows) where the conclusion is false and yet all the premises are true. This means that the argument is valid. QED.

In principle, if we have symbolized an argument correctly, we can determine its validity. Our method scales, but the exponential 2^n, where n is the number of component simple sentences, can cause it to become burdensome.

We have a partial work-around. What if we only find the truth values of the conclusion, then for those that are <u>false</u>, check out the truth values for the premises using the truth value assigments to the component sentences <u>that made the conclusion false</u>.

If one or more premises are forced to be false with these assignments, and this happens for every false value for the conclusion, then the argument is valid. Here is an example of the so-called Partial Truth Table method. There are 4 sentence variables so we have $2^4 = 16$ rows. But we only have to check two!

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				1st premise			2nd premise ∴		Conc.	
p	q	r	s	$(p \vee q)$	$\supset$	$(r \vee s)$	$p \supset$	$\neg r$	$p \supset$	$(s \vee q)$
T	T	T	T						T	T
T	T	T	F						T	T
T	T	F	T						T	T
T	T	F	F						T	T
T	F	T	T						T	T
T	F	T	F	T	$\boxed{T}$	T	$\boxed{F}$	F	F	F
T	F	F	T						T	T
T	F	F	F	T	$\boxed{F}$	F	$\boxed{T}$	T	F	F
F	T	T	T						T	T
F	T	T	F						T	T
F	T	F	T						T	T
F	T	F	F						T	T
F	F	T	T						T	T
F	F	T	F						T	T
F	F	F	T						T	T
F	F	F	F						T	T

Rows 6 & 8 are the only rows where the conclusion is <u>false</u>

In row 6, premise #2 is false

In row 8, premise #1 is false

The argument is <u>valid</u>

We can even shorten our method if we only want to prove invalidity: This is the Short Truth Table method.

p	q	r	1st premise $(p \bullet (q \bullet r))$	2nd premise $p \supset (r \vee \neg q)$	$\therefore$ Conc. $p \supset (\neg q \vee \neg r)$
T	T	T	$\boxed{T}$ T	$\boxed{T}$ T	$\boxed{F}$ F
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

both true while is false ✶

Argument is <u>invalid</u>

(first try, too)

Notice the systematic assignment of T/F to the components. This makes sure <u>every</u> combination is addressed.