Logic - Lecture 7

Rules of Replacement

There are ten common situations in which the transformation of a logical formula into an equivalent expression may offer some benefit in driving a proof forward. We know two expressions are logically equivalent if they have the same truth table.

① Double negation (DN)

$$p :: \neg\neg p$$

The four dots "::" are the "replacement sign."

② Commutation (Comm.)

$$(p \vee q) :: (q \vee p)$$

and $(p \cdot q) :: (q \cdot p)$

(3) Association (Assoc.)

$((p \vee q) \vee r) :: (p \vee (q \vee r))$

$((p \cdot q) \cdot r) :: (p \cdot (q \cdot r))$

(4) Duplication (Dup.)

$p :: p \vee p$

$p :: p \cdot p$

(5) DeMorgan (DeM)

$\neg(p \vee q) :: (\neg p \cdot \neg q)$

$\neg(p \cdot q) :: (\neg p \vee \neg q)$

(6) Biconditional Exchange (BE)

$(p \equiv q) :: ((p \supset q) \cdot (q \supset p))$

⑦ Contraposition (Contra.)

$(p \supset q) :: (\neg q \supset \neg p)$

⑧ Conditional Exchange (CE)

$(p \supset q) :: (\neg p \vee q)$

⑨ Exportation (Exp.)

$((p \cdot q) \supset r) :: (p \supset (q \supset r))$

⑩ Distribution (Dist.)

$(p \cdot (q \vee r)) :: ((p \cdot q) \vee (p \cdot r))$

$(p \vee (q \cdot r)) :: ((p \vee q) \cdot (p \vee r))$

That's the list. Remember, the :: sign means either side may be replaced

by the other.

Here is a model argument:

Premises: ① $(\neg A \vee \neg B) \supset (\neg C \vee D)$

② $\neg C \supset (E \cdot F)$

③ $E \cdot \neg (F \vee D)$

/ ∴ A

④ $\neg (F \vee D)$

⑤ $\neg F \cdot \neg D$

⑥ $\neg F$

⑦ $\neg D$

⑧ $\neg E \vee \neg F$

⑨ $\neg (E \cdot F)$

⑩ $\neg \neg C$

(11) $\neg\neg C \cdot \neg D$

(12) $\neg(\neg C \vee D)$

(13) $\neg(\neg A \vee \neg B)$

(14) $\neg\neg A \cdot \neg\neg B$

(15) $\neg\neg A$

(16) A ★