Logic - Lecture

Rules of Inference

Learn this stuff by heart.

We know the distinction between a form and a substitution instance. It is analagous to a variable vs. a constant.

A statement is a substitution instance of a statement form where other simple (or complex) statements are substituted consistently for each variable in the form.

Ex: $(p \supset q) \cdot (\neg p \supset \neg r)$

$$(R \supset W) \cdot (\neg R \supset \neg P)$$

R = It rains., W = It is wet.,

P = There are puddles.

Keep in mind each simple variable $p, q, \ldots$ may be replaced by arbitrarily complex formulas themselves. But be careful – $((A \supset B) \vee C)$ is <u>not</u> a substitution instance of $p \supset q$. If we set $p = A$ then q would be $B) \vee C$, which is not a valid formula. Major operators must align. That said, it is perfectly OK to write $A \supset A$ as a substitution instance of $p \supset q$. If any other p's or q's appear in the formula, A <u>has to</u> go in for each one.

<u>Proving Stuff</u>:

See if this sounds reasonable:

1) John is weak and has a stomach ache.

2) If he is weak and has a stomach ache, and has either a rash or a fever, then he either has food poisoning or mononucleosis.

3) He would have food poisoning only if he ate bean sandwiches at Joe's Bar and Grill last night.

4) But he didn't have bean sandwiches at Joe's last night; instead he had lobster at Maximillian's.

5) He doesn't have a rash, but he does have a fever.

6) THEREFORE... he must have mono.

Let us symbolize this argument

using obvious statement abbreviations:

1) $(W \cdot S)$

2) $((W \cdot S) \cdot (R \vee F)) \supset (P \vee M)$

3) $(P \supset B)$

4) $\neg B \cdot L$

5) $\neg R \cdot F$

$\therefore M$

We can deduce the conclusion as follows:

6) F from 5) he does have a fever

7) $(R \vee F)$ from 6) he has a rash or fever

8) $(W \cdot S) \cdot (R \vee F)$ from 1) and 7)

9) $(P \vee M)$ from 2) and 8)

10) $\neg B$ from 4)

11) $\neg P$ from 3) and 10)

12) M from 9) and 11)

This is a formal logical proof. The steps may seem clear or not so much. They are based on rules of inference onalagous to the rules of algebra in a calculation. There are eight basic rules. They have names, so when we use one we will make a note of it in our proofs.

1 Modus Ponens (MP)

$$\begin{array}{l} p \supset q \\ \underline{p \qquad} \\ \therefore\ q \end{array}$$

2 Modus Tollens (MT)

$$\begin{array}{l} p \supset q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

3 Hypothetical Syllogism (HS)

$$\begin{array}{l} p \supset q \\ q \supset r \\ \hline \therefore p \supset r \end{array}$$

4 Simplification (Simp)

$$\begin{array}{l} p \cdot q \\ \hline \therefore p \end{array} \quad \text{and} \quad \begin{array}{l} p \cdot q \\ \hline \therefore q \end{array}$$

Conjunction "commutes"

5 Conjunction (Conj)

$$\begin{array}{l} p \\ q \\ \hline \therefore p \cdot q \end{array} \longrightarrow \text{or } q \cdot p$$

6 Disjunctive Syllogism (DS)

$$\begin{array}{l} p \vee q \\ \underline{\neg p} \\ \therefore q \end{array} \quad \text{or} \quad \begin{array}{l} p \vee q \\ \underline{\neg q} \\ \therefore p \end{array}$$

Disjunction also commutes.

7 Addition (Add)

$$\begin{array}{l} \underline{\quad p \quad} \\ \therefore p \vee q \end{array} \quad \text{or} \quad \begin{array}{l} \underline{\quad q \quad} \\ \therefore p \vee q \end{array}$$

8 Constructive Dilemma (CD)

$$\begin{array}{l} p \vee r \\ p \supset q \\ \underline{r \supset s} \\ \therefore q \vee s \end{array}$$

Each rule has a truth table that justifies it.

For MP:

p	q	$p \supset q$
T	T	T
T	F	F
F	T	T
F	F	T

When p is true and $p \supset q$ is true, so is q.

Each rule is truth-preserving.

The proof process for an argument goes like this:

1) Symbolize the premises and list them in order.

2) Symbolize the conclusion and write it at the end.

3) Apply the rules of inference one at a time to justify derivation of the conclusion.

Here is a model:

If I smoke or drink too much, then I don't sleep well, and if I don't sleep well or don't eat well, then I feel rotten. If I feel rotten, I don't exercise and don't study enough. I do smoke too much. Therefore, I don't study enough.

Let: S = smoke too much
D = drink too much
P = sleep well
E = eat well
R = feel rotten
X = exercise
T = study enough

1) $(S \vee D) \supset \neg P$

2) $(\neg P \vee \neg E) \supset R$

3) $R \supset (\neg X \cdot \neg T)$

4) S

/ $\therefore \neg T$

derived formula	justification
5) $S \vee D$	(4; Add)
6) $\neg P$	(1, 5; MP)
7) $\neg P \vee \neg E$	(6; Add)
8) R	(2, 7; MP)
9) $\neg X \cdot \neg T$	(3, 8; MP)
10 $\neg T$	(9; Smp) $\blacksquare$