LOGIC - LECTURE 3

We know the characteristics of our five sentential operators - truth-functional connectives. We would like to symbolize arbitrary sentences containing them as well as non-truth functional operators. There are many words in English that convey the meaning of our operators in less-than-obvious ways. Here is a discussion of the main ones.

1) Conjunction - "and" - "$\cdot$"

a) "but"

Ex. John likes TV <u>but</u> Mary hates it.

This sentence claims both components are true. An obvious symbolization would be $J \cdot \neg M$

b) "however"

Ex. John likes TV <u>however</u> Mary hates it.

c) "nevertheless"

Ex. You see the pattern:

John likes TV $\boxed{\quad\quad}$ Mary hates it.

d) "still" - used as a connective, not an adverb

e) "although"

f) "even though"

g) " ; " - semi-colon

h) "while"

i) "despite"

j) "moreover"

2) Disjunction - "or" - "V"

a) "either...or"

b) "or else"

Disjunction is easy to recognize but remember "V" means either or both (weak disjunction).

"You may have soup or salad"

soup salad

really means $(P \vee D) \bullet (\neg(P \bullet D))$

not both

3) Negation - "not" - "$\neg$" or "$\sim$"

"Not" can be tricky. Recall our simple sentences (which receive a letter abbreviation) are <u>positive</u> by convention. If "J" = "John likes TV", then we would signify "John hates TV" as $\neg J$ and not some new symbolization, say N.

Sometimes it is not clear in a sentence if the entire sentence is being negated, or just the predicate.

Suppose "L" = "John is lucky".
Then "¬L" means "John is not lucky"...
perhaps he does not win lotteries.
To say "John is unlucky" seems
to imply something stronger... he
has no luck at all. So we might
symbolize that as "U". It is not
cut-and-dried. Use judgment.

a) "neither" is a negated disjunction.
Neither = "not either". If neither
Al or Bob won, it means two things:
(Al didn't win) and (Bob didn't win)

b) "not both" is a negated conjunction

Now is the time to discuss De Morgan's Roles :

(i) Negation of Disjunction is equivalent to the Conjunction of Negations :

$$\neg(A \vee B) \equiv \neg A \cdot \neg B$$

(ii) Negation of Conjunction is equivalent to the Disjunction of Negations :

$$\neg(A \cdot B) \equiv \neg A \vee \neg B$$

c) "it is not the case that..."

d) "it is false that..."

4) Conditional - "if...then" - "$\supset$" or Material Implication

a) if A, then B $\quad A \supset B$

b) if A, B $\quad A \supset B$

c) B, if A $\quad A \supset B$

d) B, provided A $\quad A \supset B$

e) provided A, then B $\quad A \supset B$

f) B only if A $\quad B \supset A$

g) B only if A $\quad \neg A \supset \neg B$

h) A only if B $\quad A \supset B$

i) A only if B $\quad \neg B \supset \neg A$

j) A unless B $\quad \neg B \supset A$

k) A unless B $\quad A \vee B$

l) ¬A unless B $\quad \neg B \supset \neg A$

m) ¬A unless B $\quad \neg A \vee B$

5) Biconditional - "if and only if" - "$\equiv$"

In math this is written as "iff".

Remember $A \equiv B$ means both $A \supset B$ <u>and</u> $B \supset A$.

a) "necessary and sufficient"

A necessary for B means $B \supset A$

A sufficient for B means $A \supset B$

b) "when(ever) and only when(ever)"

A whenever B means $B \supset A$

A only whenever B means $A \supset B$

MISC:

A not without B means $A \supset B$

B follows from A means $A \supset B$

B is implied by A means $A \supset B$

A is equivalent (materially) to B means $A \equiv B$.