# LOGIC - LECTURE 2.3

## TRUTH VALUES -

A truth function maps sentences to the set {True, False}.

We would like to know how our sentential operators affect formulas with various true and false sentences in them. It is fortunate that our sentential operators are truth-functional connectives. This means the truth value of two formulas connected by an operator is completely determined by the individual truth values of the component formulas.

For example, if A is true and B is true, then A·B is true:

"John is tall and Mary is rich" is true is John truly is tall and Mary truly is rich. If John were not tall or Mary were not rich, or perhaps both, then the conjunction statement would be false. We summarize the possibilities as follows, with the obvious symbolization

T = true
F = false

| J | M | J·M |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

This is a truth table. It basically defines "•" by its behavior with respect to truth values.

What if we stated:

"Either John is tall or Mary is rich".

Then the table would be:

| J | M | J v M |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

You can see that "or" is more forgiving with respect to conserving truth.

Negation is easy:

| $J$ | $\neg J$ |
|---|---|
| T | F |
| F | T |

Let's add a new statement for implication: "John will make the basketball team". Symbolize it with "B." Consider $J \supset B$ (if John is tall he will make the basketball team). The truth table looks like this:

| | $J$ | $B$ | $J \supset B$ |
|---|---|---|---|
| 1) | T | T | T |
| 2) | T | F | F |
| 3) | F | T | T |
| 4) | F | F | T |

The $\supset$ operator is tricky. We call it "material implication". It is a weak form of implication. Certainly lines 1) and 2) in the table capture our usual understanding. If John is tall <u>and</u> makes the team, $J \supset B$ is true. If he is tall and does <u>not</u> make the team, then $J \supset B$ is false. If he is not tall and still makes the team (line 3)) is the claim that <u>if</u> he is tall he will make the team incorrect? No, it doesn't apply. Like-wise for not tall/didn't make team (line 4))

I told you it was weak implication.

What could lines 3 & 4 be in any case?

| | J | B | (i) | (ii) | (iii) | (iv) |
|---|---|---|---|---|---|---|
| 3) | F | T | T | F | T | F |
| 4) | F | F | T | F | F | T |

Column (i) is what we are going to use. Note column (ii) would replicate the truth table for conjunction. Also (iii) just returns the truth values for B. And (iv) turns out to represent the biconditional "≡". We are forced into this.

Here are some handy rules:

1. If one disjunct is true, the entire disjunction is true
2. If one conjunct is false, the entire conjunction is false
3. If the antecedent is false, the conditional is true
4. If the consequent is true, the conditional is true.

Now if we have A & B true and X & Y false, what is the truth value of $(A \cdot X) \vee (B \cdot Y)$?

Using our tables or handy rules,

$A \cdot X$ is false, $B \cdot Y$ is also false,

so $(A \cdot X) \vee (B \cdot Y)$ is false.

How about a connective that is <u>not</u> truth functional?

S = "I got sick"

A = "I ate the sandwich"

S#A joins S and A with

# = "because". Knowing the truth of S and A sheds no light on S#A.

Maybe it* is true, maybe not.