Logic - Lecture 15

Pure Quantifier Arguments

Let us analyze arguments where all the premises and the conclusion are quantifier statements or their negations. Here is a typical argument that uses U.G. One thing to keep in mind with arguments having universal conclusions is that the propositional function of the conclusion will often be a conditional. So it is common to use CP (conditional proof) to derive an instance which can then be generalized.

1)	$(x)((Fx \vee Gx) \supset \neg(Hx \vee Ix))$	Premise
2)	$(x)(Zx \supset (Hx \cdot Wx))$	Premise
/∴	$(x)(Gx \supset \neg Zx)$	Conclusion
3)	flag a	F.S. (UG)
4)	$(Fa \vee Ga) \supset \neg(Ha \vee Ia)$	1; UI
5)	$Za \supset (Ha \cdot Wa)$	2; UI
6)	Ga	Assum; CP
7)	$Fa \vee Ga$	6; Add
8)	$\neg(Ha \vee Ia)$	4, 7; MP
9)	$\neg Ha \cdot \neg Ia$	8; DeM
10)	$\neg Ha$	9; Simp
11)	$\neg Ha \vee \neg Wa$	10; Add
12)	$\neg(Ha \cdot Wa)$	11; DeM

UG subproof marker

CP subproof marker

13)	$\neg Za$	5,12; MT
14)	$Ga \supset \neg Za$	6-13; CP
15)	$(x)(Gx \supset \neg Zx)$	14; UG

Note you can never end a CP on the same line as a UG. Another tip: you should not start a UG subproof after already instantiating the letter you plan to generalize on... if you do an EI before a UG, you will be flagging the EI letter and then you can't properly re-use it in the UG.

Here is an example using negated quantifiers:

1) $\neg(x)(Fx \supset (Gx \vee Hx))$ Premise

2) $(x)(Ax \supset Gx)$ Premise

/ ∴ $\neg(x)(Fx \supset Ax)$ Conclusion

3) $(\exists x)(Fx \cdot \neg(Gx \vee Hx))$ CQN; 1

get rid of $\neg(x)$ ... now with (3)

4) $Fa \cdot \neg(Ga \vee Ha)$ flag a 3; EI

5) $Aa \supset Ga$ 2; UI

6) Fa 4; Simp

7) $\neg(Ga \vee Ha)$ 4; Simp

8) $\neg Ga \cdot \neg Ha$ 7; DeM

9) $\neg Ga$ 8; Simp

10) $\neg Aa$ 5, 9; MT

11) $Fa \cdot \neg Aa$ 6, 10; Conj

12) $(\exists x)(Fx \cdot \neg Ax)$ 11; EG

13) $\neg(x)(Fx \supset Ax)$ 12; CQN

Note that when the conclusion is a negated quantifier statement, the strategy is to drive the proof to an equivalent positive quantifier statement, then use CQN at the end to flip it.

In general these proofs have the following structure:

1) Use QN or CQN to create positive statements if necessary
2) Use instantiation
3) Use sentential rules to develop instances
4) Generalize with UG or EG
5) Use QN or CQN as needed to get conclusion

Remember - once you discharge an assumption or a flagging step, you can not come back and use the assumption or any step in the flagged subproof again.

Try this one (most famous proof from Aristotle)

1) All men are mortal $(x)(Mx \supset Rx)$

2) Socrates is a man Ms ?

/∴ Socrates is mortal Rs