Logic - Lecture 14

## Quantifier Rules - Preliminary Form

We are going to describe four rules, two in which we pass from the general to the particular, and two from the particular to the general. The two rules mentioned in the last lecture require no additional restrictions, so they may be considered in final form:

UI (universal instantiation)

$$(x)\phi x$$
$$/\therefore \phi a$$

and

EG (existential generalization)

$$\phi a$$
$$/\therefore (\exists x)\phi x$$

The other two:

UG (universal generalization)

→ flag $a$

$\phi a$

---

$/\therefore (x)\phi x$

and

EI (existential instantiation)

$(\exists x)\phi x$ – flag $a$

---

$/\therefore \phi a$

require some explanation.

The business of "flagging" is a method of tracking individual named objects so that we don't mistakenly assume properties that are not justified. A quick example of what can happen:

EI {There exists an animal that is a cat.
Fluffy is a cat.
Fluffy is a dog. (some <u>other</u> Fluffy)
Fluffy is both a dog and a cat.

The second Fluffy should be called Fluffy2.

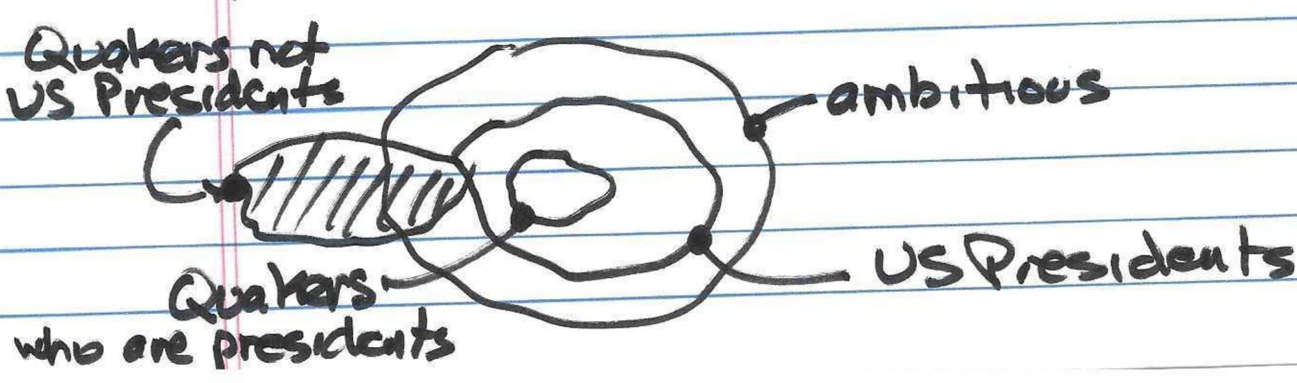
Then the conjunction is "Fluffy is a cat and Fluffy2 is a dog."

Here is a model argument.

(1) All U.S. presidents have been ambitious.

(2) Some U.S. presidents have been Quakers.

/∴ Some Quakers have been ambitious.



We see this is true. Here is a proof.

1. $(x)(Px \supset Ax)$ premise
2. $(\exists x)(Px \cdot Qx)$ premise

/ $\therefore (\exists x)(Qx \cdot Ax)$ conclusion

3. $Pa \cdot Qa$ EI; 2
4. $Pa \supset Aa$ UI; 1
5. $Pa$ Simp; 3
6. $Aa$ MP; 4,5
7. $Qa$ Simp; 3
8. $Qa \cdot Aa$ Conj; 6,7
9. $(\exists x)(Qx \cdot Ax)$ EG; 8

The pattern here is to first instantiate form what we want, then generalize.

Note that when we instantiate $\phi x$, <u>every</u> occurrence of x is replaced with the <u>same</u> constant. $\phi a$ is an <u>instance</u> of $(x)\phi x$ or $(\exists x)\phi x$. No other constants or variables are changed in this process.

OK, now the details about EI and UG:

Argument: There exists a number which is both odd and even (think Fluffy).

1. $(\exists x)(Ox \cdot Nx)$ premise
2. $(\exists x)(Ex \cdot Nx)$ premise
3. $Oa \cdot Na$ EI; 1
4. $Ea \cdot Na$ EI; 2
5. $Oa$ Simp; 3

6. Oa · Ea · Na  Conj; 4,5

7. $(\exists x)(Ox \cdot Ex \cdot Nx)$ EG; 6

Where is the mistake? Line 4.

Every EI has to introduce a new constant. Line 4 should have read Eb · Nb / EI; 2. We make this explicit when we use an EI:

1. $(\exists x)(Ox \cdot Nx)$ premise
2. $(\exists x)(Ex \cdot Nx)$ premise
3. Oa · Na  EI; 1 (flag a)

now a cannot be used in any further EI.

Here is a correct proof with EI flagging.

1. $(\exists x)(Fx \cdot \neg Gx)$ Premise
2. $(x)(Hx \supset Gx)$ Premise

$/ \therefore (\exists x)(Fx \cdot \neg Hx)$ Conclusion

3. $Fa \cdot \neg Ga$ EI; 1 <u>flag a</u>
4. $Ha \supset Ga$ UI; 2
5. $Fa$ Simp; 3
6. $\neg Ga$ Simp; 3
7. $\neg Ha$ MT; 4, 6
8. $Fa \cdot \neg Ha$ Conj; 5, 7
9. $(\exists x)(Fx \cdot \neg Hx)$ EG; 8

OK... that's how EI works. Don't re-use a flagged constant in another EI! and you are good to go.

Universal ~~Instantiation~~ Generalization is the fussiest rule. To guarantee that we have a genuine universal instance, from which it is OK to infer a universal proposition, we enact the rule that the instance for a UG be derived within a special kind of subproof called a "flagged subproof".

This subproof will keep the instance we plan to generalize from contamination from existential or contingent statements.

We indent the subproof as we did for conditional and indirect proofs.

Here is a model argument:

All cats are mammals. All mammals are vertebrates. Therefore all cats are vertebrates.

| | | |
|---|---|---|
| 1 | $(x)(Cx \supset Mx)$ | Premise |
| 2 | $(x)(Mx \supset Vx)$ | Premise |
| / | $\therefore (x)(Cx \supset Vx)$ | Conclusion |

indent more

| | | |
|---|---|---|
| 3 | flag a | FS (UG) "flagged subproof" |
| 4 | $Ca \supset Ma$ | UI; 1 |
| 5 | $Ma \supset Va$ | UI; 2 |
| 6 | $Ca \supset Va$ | HS; 4,5 |
| 7 | $(x)(Cx \supset Vx)$ | UG; 6 |

this is an "a-flagged scope marker"

The UG in line 7 "discharges" the subproof.

You can use other types of subproofs with flagged subproofs:

| | | |
|---|---|---|
| 1 | $(x)((Fx \vee Gx) \supset Hx)$ | Premise |
| 2 | $(x)(Hx \supset (Ix \cdot Sx))$ | Premise |
| | $/ \therefore (x)(Fx \supset Sx)$ | Conclusion |
| 3 | flag a | FS (UG) |
| 4 | $Fa$ | Assumption (CP) |
| 5 | $(Fa \vee Ga) \supset Ha$ | UI; 1 |
| 6 | $Ha \supset (Ia \cdot Sa)$ | UI; 2 |
| 7 | $Fa \vee Ga$ | Add; 4 |
| 8 | $Ha$ | MP; 5, 7 |
| 9 | $Ia \cdot Sa$ | MP; 6, 8 |
| 10 | $Sa$ | Simp; 9 |
| 11 | $Fa \supset Sa$ | CP; 4–10 |
| 12 | $(x)(Fx \supset Sx)$ | UG; 11 |

Here is a short argument that is wrong.

some (1) $(\exists x)(Cx \cdot Sx)$ Premise

(2) $\rightarrow$ Flag a FS(UG)

(3) $Ca \cdot Sa$ EI; 1, flag a

all (4) $(x)(Cx \cdot Sx)$ UG; 3

What went wrong. "a" was flagged in step 2, then re-used (re-flagged) in step 3.

★ A flagged letter should not appear outside the subproof that flags it.