Logic - Lecture 13

Quantifier Form

This lecture is about notation.

Definition: A statement is in *quantifier form* if:

1. It begins with a quantifier
2. The scope of the initial quantifier extends to the end of the line (formula)

Formally, quantifier statements look like $(x)\phi x$ or $(\exists x)\psi x$ where ϕ and ψ are formulas entirely within the scope of the initial universal or existential quantifiers.

Some examples:

1. $(\exists x)(Fx \cdot Gx)$ is in QF
2. $(\exists x)Fx \cdot (\exists x)Gx$ is not in QF

Let F = "is a dog" and G = "is a cat" to see the difference.

We often have quantified statements connected with truth-functional symbols:

$(\exists x)Fx \cdot (\exists x)Gx$

$(x)(Fx \supset Gx) \vee (\exists x)(Fx \supset \neg Gx)$

$(\exists x)Fx \supset (x)(Gx \vee Hx)$

$(x)Fx \equiv (y)Gy$

These are perfectly fine quantified statements, they are just not in quantifier form.

Remember that negated quantifier statements are not in quantifier form. They don't begin with either (x) or $(\exists x)$, but rather "$\neg$".

The meanings can be a little tricky. Consider $(\exists x)Fx \supset (\exists x)Ux$.

Let F = "is a fish" and U = "is a unicorn".

This statement is false, since a fish exists and unicorns do not. (True $\supset$ False)

But $(\exists x)(Fx \supset Ux)$ is true, just let x be a tree, then False $\supset$ False is true!

The next homework will ask you to analyze English sentences and determine if it is in quantifier form or not.

Why do we care about this? Because we are about to have four rules that will allow us to construct predicate calculus proofs. The rules are very specific as to what they allow, and they are stated in terms only applicable to statements in quantifier form.

Two rules are very simple, so here is a preview:

1. Universal Instantiation (UI)

$$\frac{(x)\,\phi x}{/\therefore\ \phi a}\quad \begin{matrix}\leftarrow \text{general} \\ \leftarrow \text{specific/particular}\end{matrix}$$

2. Existential Generalization (EG)

$$\frac{\phi a}{/\therefore\ (\exists x)\,\phi x}\quad \begin{matrix}\leftarrow \text{particular} \\ \leftarrow \text{general}\end{matrix}$$