Logic - Lecture 12.

Complex Predicates & Subjects

This subject is an easy and straightforward

extension of what we already know.

Suppose we have the following

categorical statement:

$$\neg (x)((Fx \cdot \neg Gx) \supset (Hx \cdot \neg Ix))$$

How can this be transformed into

equivalent statements using our CQN

and other rules? Some examples:

① $(\exists x)((Fx \cdot \neg Gx) \cdot \neg (Hx \cdot \neg Ix))$

② $(\exists x)((Fx \cdot \neg Gx) \cdot (\neg Hx \vee Ix))$

③ $(\exists x)((Fx \cdot \neg Gx) \cdot (Hx \supset Ix))$

④ $(\exists x)((Fx \cdot \neg Gx) \cdot (\neg Ix \supset \neg Hx))$

Or this one:

$$\neg (x)\left(\left(Fx \cdot (Gx \vee Hx)\right) \supset \left((Px \cdot Qx) \supset \neg Rx\right)\right)$$

Some equivalents:

① $(\exists x)\left(\left(Fx \cdot (Gx \vee Hx)\right) \cdot \neg \left((Px \cdot Qx) \supset \neg Rx\right)\right)$

② $(\exists x)\left(\left(Fx \cdot (Gx \vee Hx)\right) \cdot \neg \left(\neg (Px \cdot Qx) \vee \neg Rx\right)\right)$

etc.

Let's start with a universal negative

(Type E) English sentence.

"No Siamese cat that is well-fed is

either quiet or unfriendly".

The complex subject is "Siamese cat

that is well-fed". The complex predicate

is "quiet or unfriendly". With

obvious symbolization, the subject is

$((Sx \cdot Cx) \cdot Wx)$. The predicate becomes $(Qx \lor \neg Fx)$. The overall form of the statement is $\neg (\exists x)(\underline{subj} \cdot \underline{pred})$.

Putting it all together:

$$\neg (\exists x)(((Sx \cdot Cx) \cdot Wx) \cdot (Qx \lor \neg Fx)).$$

IOW, there does not exist an x such that x is a cat, x is Siamese, x is well-fed and x is either quiet or unfriendly.

Here is another: "No English sheepdog is vicious unless it is either mistreated or ill." This statement has pattern "No A is a B unless it is a C."

We can code this as a universal

as follows : $(x)(Ax \supset (\neg Cx \supset \neg Bx))$.

IOW, for all x, if x is an A, then if x is not C it is also not B.

Back to the English sentence (and dog):

The subject is "English sheepdog".

The predicate is "not vicious unless mistreated or ill." With obvious abbreviations:

$(x)((Sx \cdot Ex) \supset (\neg (Mx \vee Ix) \supset \neg Vx))$.

So for all x, if x is a sheepdog and x is English, then if x is neither mistreated nor ill, x is not vicious.

Reworking this using equivalences:

① $\neg (\exists x)((Sx \cdot Ex \cdot Vx) \cdot \neg (Mx \lor Ix))$

② $(x)((Sx \cdot Ex) \supset (Vx \supset (Mx \lor Ix)))$

and many more.

The next homework will give you some

practice.