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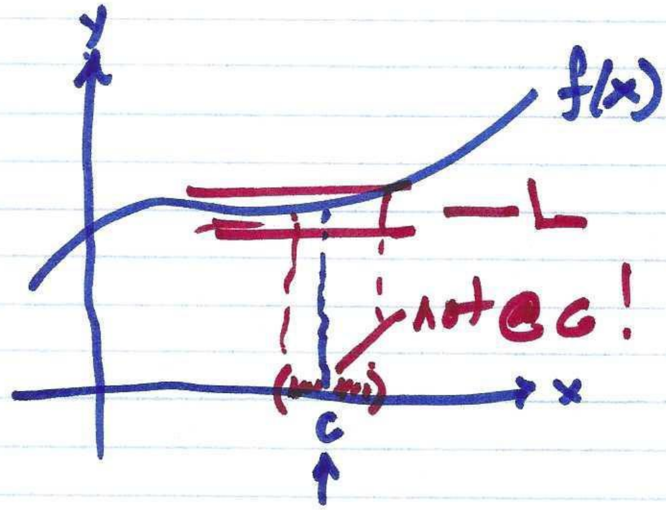
f(x) has a limit at the point $x = c$

$\lim_{x \to c} f(x) = L$

BASIC IDEA OF LIMIT



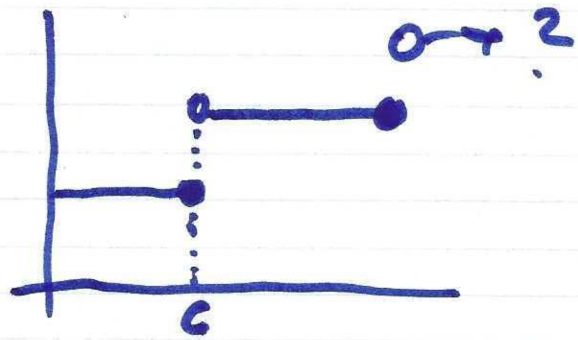
if there is a value L (the limit)

such that, if we are arbitrarily close to c (on the x-axis) ... but not @ c ...

then f(x) is arbitrarily close to L

Related idea:

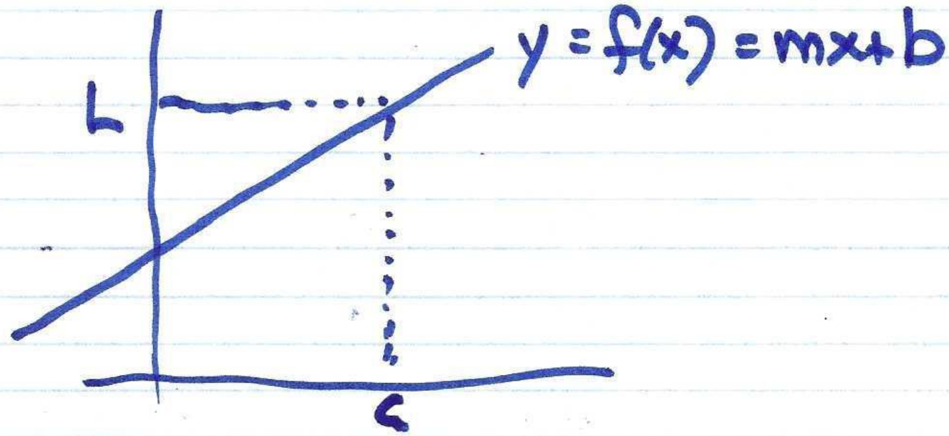
Limits from right or left



$\lim_{x \to c^-}$ can exist & be different than $\lim_{x \to c^+}$

Some functions <u>always</u> attain their limits:

1) $f(x) = mx + b$



$\underline{L = mc + b}$ exists

2) Any polynomial

$f(x) = x^2 + 2$

$\lim_{x \to c} f(x) = c^2 + 2$

3) Any exponential

$f(x) = e^{2x+5}$

$f(c) = e^{2c+5}$

1) Any logarithmic function

$f(x) = \ln(x+7)$

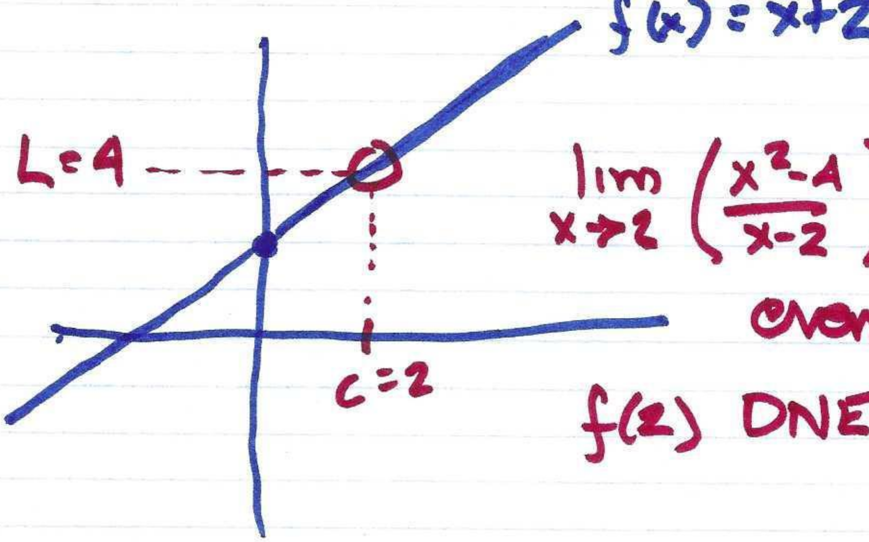
$\lim_{x \to c} f(x) = \ln(c+7)$

What about $f(x)$ does not exist @ c ....

BUT $\lim_{x \to c} f(x)$ does?

Consider $f(x) = \dfrac{x^2-4}{x-2}$; let $c = 2$

Note $\dfrac{x^2-4}{x-2} = x+2$



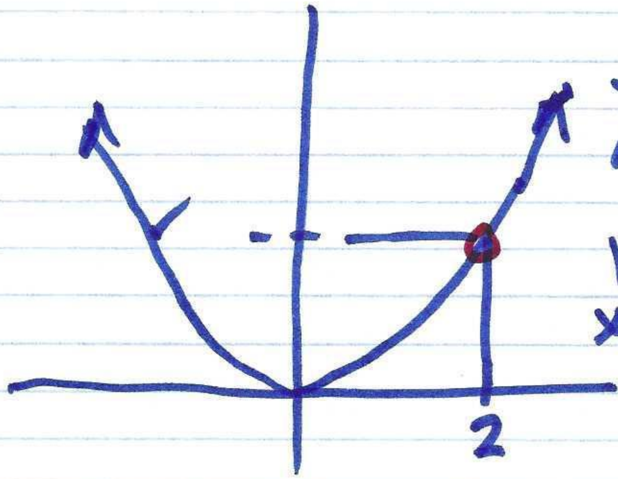
$\lim_{x \to 2} \left(\dfrac{x^2-4}{x-2} \right) = 4$

even though

$f(2)$ DNE

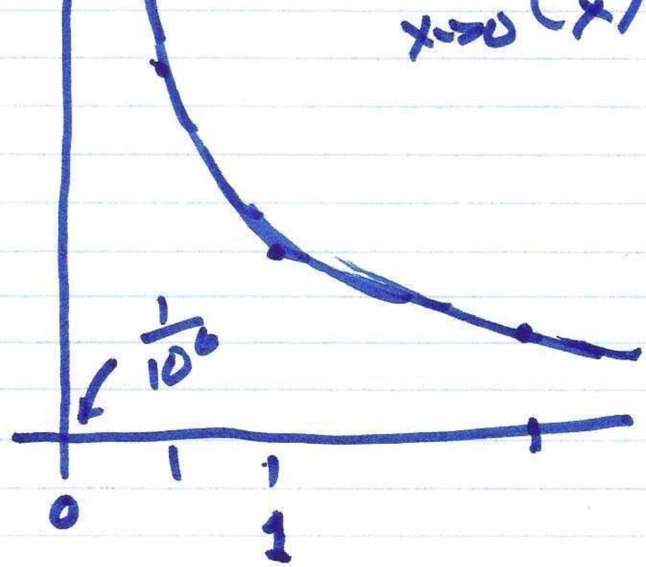
A

Consider $f(x) = \dfrac{x^3 - 2x^2}{x-2} = \dfrac{x^2\cancel{(x-2)}}{\cancel{x-2}} = x^2$



$\lim\limits_{x \to 2} f(x) = 4$!

$\lim\limits_{x \to 0} \dfrac{1}{x}$



$\lim\limits_{x \to 0} \left(\dfrac{1}{x}\right) = +\infty$

$-\infty$

Rules for Limits

Given $\lim_{x \to c} f(x) = L$

and $\lim_{x \to c} g(x) = M$

1) $\lim_{x \to c} \left(f(x) + g(x) \right) = L + M$

2) $\lim_{x \to c} \left(f(x) - g(x) \right) = L - M$

3) $\lim_{x \to c} \left(f(x) \cdot g(x) \right) = L \cdot M$

4) $\lim_{x \to c} \left(\frac{f(x)}{g(x)} \right) = \frac{L}{M} \quad (M \neq 0)$

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5) $\lim_{x \to c} \left(r f(x) \right) = rL$

6) Polynomial Rule

If $p(x)$ is a polynomial

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots a_1 x + a_0$$

$$\lim_{x \to c} \left(p(x) \right) = p(c)$$

7) Power Rule

For any K, $\lim_{x \to c} \left[f(x) \right]^K = \left[\lim_{x \to c} f(x) \right]^K = L^K$

Ex: $\lim_{x \to 2} \left(\underbrace{x^2 - 2x + 1}_{(x-1)^2} \right)^5 = \lim_{x \to 2} \left(x - 1 \right)^{10} = 1^{10} = 1$

8) Exponent Rule

For $b > 0$, $\lim_{x \to c} \left(b^x \right) = b^c$

9) Logarithm Rule

$$\lim_{x \to c} \left(\log_b f(x) \right) = \log_b \left(\lim_{x \to c} f(x) \right) = \log_b L$$

Prob $\lim_{x \to 1} \frac{x^2-2x+1}{(x-1)^3} = \frac{(x-1)^2}{(x-1)^3} = \frac{1}{x-1}$

negative for $x<1$
positive " $x>1$

So left limit is $-\infty$
right " " $+\infty$ Limit DNE.

Prob $f(t) = \frac{12t^2-15t+12}{t^2+1} \sim \frac{12t^2}{t^2} = \underline{\underline{12}}$

Find $\lim_{t \to \infty} f(t) = 12$