$f'(x)$ denotes derivative

$y = f(x)$ $\frac{dy}{dx}$ $\frac{d}{dx}[f(x)]$ $D_x[f(x)]$

Recall defⁿ of $f'(x)$: Δx is increment

$$\lim_{\Delta x \to 0} \left(\frac{f(x+\Delta x) - f(x)}{\Delta x} \right)$$

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① Let $f(x) = c$, constant

$$\lim_{\Delta x \to 0} \left(\frac{f(x+\Delta x) - f(x)}{\Delta x} \right) = \lim_{\Delta x \to 0} \left(\frac{c - c}{\Delta x} \right) = 0$$

② Let $f(x) = x$

$$\lim_{\Delta x \to 0} \left(\frac{(x+\Delta x) - x}{\Delta x} \right) = \lim_{\Delta x \to 0} \left(\frac{\Delta x}{\Delta x} \right) = 1$$

3 Let $f(x) = x^2$

$$f'(x) = \lim_{\Delta x \to 0} \left(\frac{(x+\Delta x)^2 - x^2}{\Delta x} \right) =$$

$$= \lim_{\Delta x \to 0} \left(\frac{x^2 + 2x\cancel{\Delta x} + (\Delta x)^{\cancel{2}} - x^2}{\cancel{\Delta x}} \right)$$

$$= \lim_{\Delta x \to 0} \left(2x + \Delta x \right) = 2x$$

4 Let $f(x) = x^3$

$$f'(x) = \lim_{\Delta x \to 0} \left(\frac{(x+\Delta x)^3 - x^3}{\Delta x} \right) =$$

$$\lim_{\Delta x \to 0} \left(\frac{\cancel{x^3} + 3x^2\cancel{\Delta x} + 3x(\Delta x)^{\cancel{2}} + (\Delta x)^{\cancel{3}} - \cancel{x^3}}{\cancel{\Delta x}} \right)$$

$$\lim_{\Delta x \to 0} \left(3x^2 + 3x\Delta x + (\Delta x)^2 \right) = 3x^2$$

$(x)' \to 1x^0 = 1 \quad (x^2)' = 2x^{2-1} \quad (x^3)' = 3x^{3-1} = \underline{3x^2}$

5 $f(x) = \frac{1}{x} = x^{-1}$

$$f'(x) = \lim_{\Delta x \to 0} \left(\frac{\frac{1}{x+\Delta x} - \frac{1}{x}}{\Delta x} \right) \leftarrow$$

$$f'(x) = (-1)x^{-1-1} = -\frac{1}{x^2}$$

6 $f(x) = \sqrt{x} = x^{1/2}$

$$f'(x) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

7 $f(x) = x^{\pi}$ $\quad f'(x) = \underline{\pi x^{\pi-1}}$

8 $f(x) = c \cdot g(x)$

$$\lim_{\Delta x \to 0} \left(\frac{cg(x+\Delta x) - cg(x)}{\Delta x} \right) = c \cdot \lim_{\Delta x \to 0} \left(\frac{g(x+\Delta x) - g(x)}{\Delta x} \right)$$

$$f'(x) = c \cdot g'(x)$$

$g'(x)$

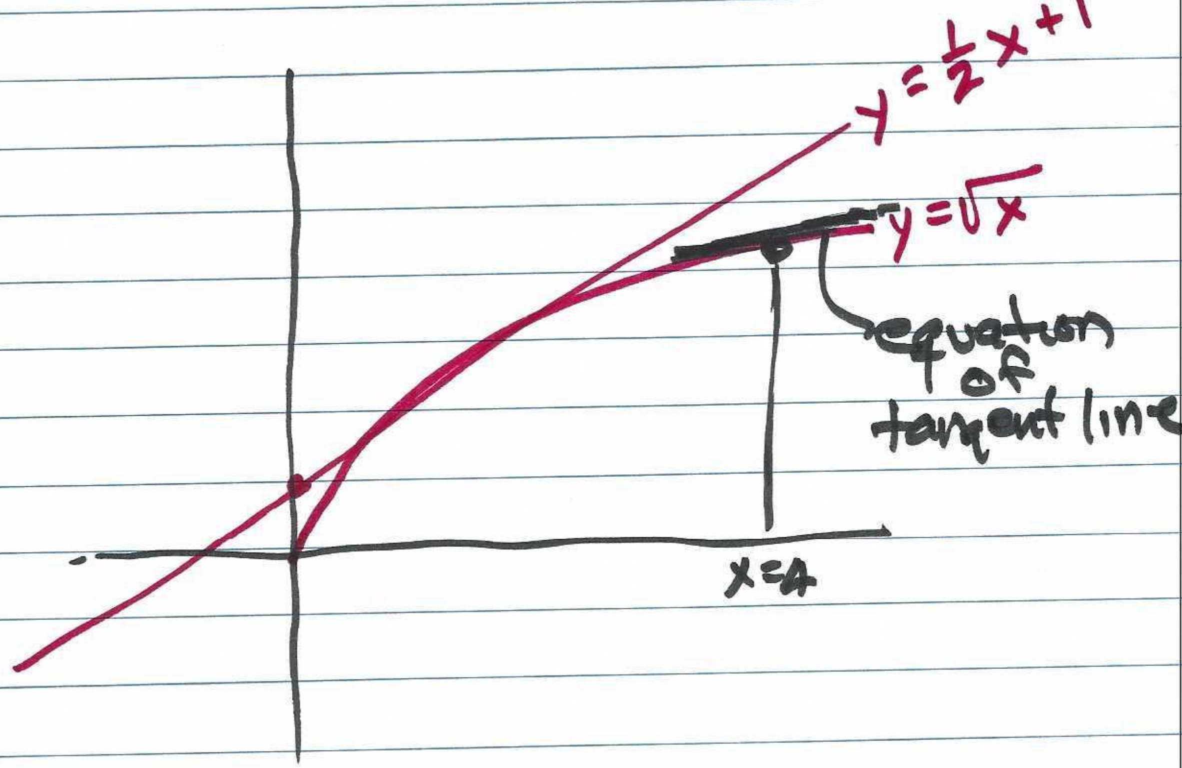
$$\textcircled{4}$$

$$\textcircled{a}\quad \left[f(x) \pm g(x)\right]' = \lim_{\Delta x \to 0}\left(\frac{f(x+\Delta x) \pm g(x+\Delta x) - (f(x) \pm g(x))}{\Delta x}\right)$$

$$= \lim_{\Delta x \to 0}\left(\frac{f(x+\Delta x - f(x)}{\Delta x}\right) \pm \lim_{\Delta x \to 0}\left(\frac{g(x+\Delta x) - g(x)}{\Delta x}\right)$$

$$= f'(x) \pm g'(x)$$

Derivative @ x is slope of tangent @ x



slope of tangent (to $\sqrt{x}$) is $\frac{1}{2\sqrt{x}}$

@ $x = 4$ slope is $\frac{1}{4}$

point of tangency is $(4,2)$

$y = mx + b \Rightarrow y = \frac{1}{4}x + b$

$1/4$

$2 = \frac{1}{4} \cdot 4 + b$

$2 = 1 + b \Rightarrow b = 1$

So tangent line has equation $y = \frac{1}{4}x + 1$

Marginal Analysis

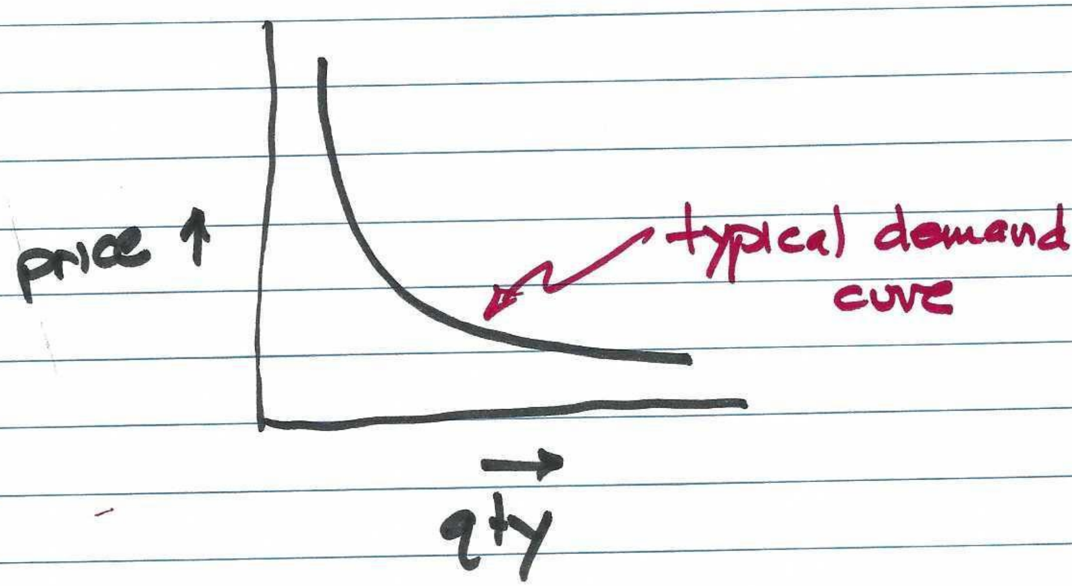
Ex 1: $C(x) = 0.4x^2 + 10x + 50$

MC $\quad C'(x) = 0.8x + 10$

So what is MC @ $x = 60$

$C'(60) = 0.8(60) + 10 = 58$

The price/quantity



$$p(q) = \frac{50,000 - q}{25,000}$$

$$2 = \frac{50,000 - q}{25,000} \qquad \underline{q = 0}$$

$$\frac{1}{2} = \frac{50,000 - q}{25,000} \qquad 37,500$$

To get revenue Rev = price · qty

$$R(q) = \left(\frac{50{,}000 - q}{25{,}000}\right) q$$

$$= 2q - \frac{q^2}{25{,}000}$$

$$R'(q) = 2 - \frac{2q}{25{,}000}$$

So @ $q = 10{,}000$ $R'(10{,}000) = 2 - \frac{4}{5} = 1.2$