9/17

Exam 9/19 - same format as last time

Review of Ch 3:

Limits:

What exactly is a limit?

Given some formula (function) $f(x)$, we say $f(x)$ approaches the limit L as x approaches some fixed value x_0 <u>if</u> we can make $|f(x) - L|$ arbitrarily small if we also make the distance $|x - x_0|$ small, <u>but not zero</u>.

Write it $\lim_{x \to x_0} f(x) = L$

1. If $\lim_{x \to x_0} f(x) = L$ and $\lim_{x \to x_0} g(x) = M$

   then $\lim_{x \to x_0} (f(x) + g(x)) = LM$ false

   $= L + M$ true

2. If $\lim_{x \to x_0} f(x) = L$ and $\lim_{x \to x_0} g(x)$ does not exist

   is it possible for $\lim_{x \to x_0} (f(x) + g(x))$ to exist?

   <u>no</u>

3 If $\lim_{x \to x_0} f(x)$ exists and $\lim_{x \to x_0} g(x)$ exists

is it always true that $\lim_{x \to x_0} \frac{f(x)}{g(x)}$ exists?

no, $\div 0$

4 Does $\lim_{x \to x_0} \left(\frac{3x^2 + 2x - 1}{x^2 + 1} \right)$ exist for all

real numbers x_0? yes, $x^2 + 1 \geq 1$

Continuity

What is continuity?

Three defining properties of continuity:

1) limit exists, i.e. $\lim_{x \to x_0} f(x) = L$

2) function value exists, i.e. $f(x_0)$ defined

3) $f(x_0) = L$ must happen

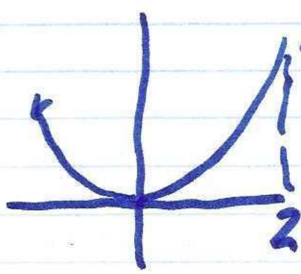
5 Is $f(x) = \frac{x^2 - 4}{x - 2}$ continuous @ $x = 2$?

no

$f(x) = x + 2$ if $x \neq 2$
not def if $x = 2$

(6) If $f(x)$ is continuous @ x_0, does $\lim_{x \to x_0} f(x)$ exist? yes, built into def^n

(7) If $h(x) = \begin{cases} x^2, & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$, is $h(x)$ continuous?

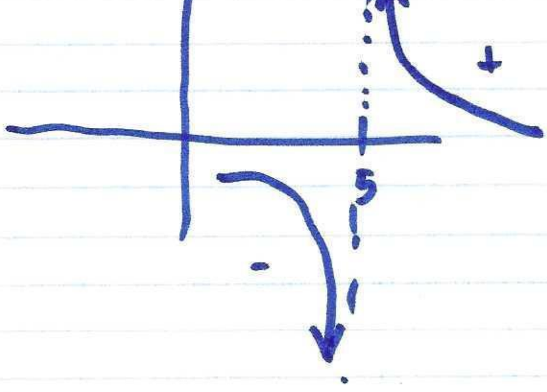


$h(2) = 1$ not equal

$\lim_{x \to 2} h(x) = 4$ so not cont.

(8) $\lim_{x \to x_0} f(x) = +\infty$ Consider $f(x) = \frac{1}{x-5}$

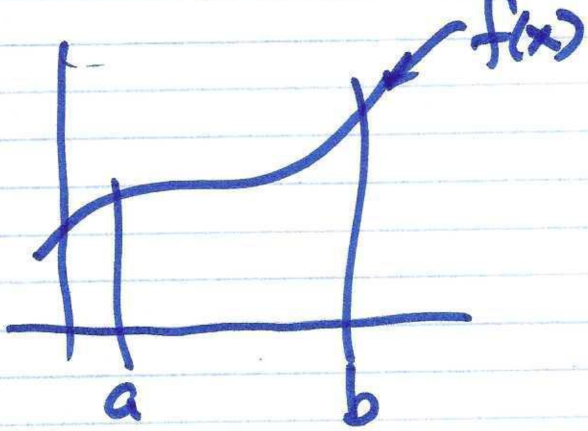
$\lim_{x \to 5^+} f(x) = +\infty$ $\lim_{x \to 5^-} f(x) = -\infty$



no limit

Average Rate of Change

$f(x)$ defined on $[a, b]$



$\Delta x = b - a$

$\Delta f = f(b) - f(a)$

So... $\bar{f}_{[a,b]} = \dfrac{\Delta f}{\Delta x} = \dfrac{f(b) - f(a)}{b - a}$

9. Is average rate of change always the instantaneous rate of change **no**

10. If $f(x)$ is increasing on $[a, b]$, is the avg. rate of change negative, i.e. $\bar{f} < 0$

no, it is > 0

Average Speed :

$$\bar{s} = \frac{d(b) - d(a)}{b-a} \rightsquigarrow \frac{\text{total dist}}{\text{total time}}$$

Instantaneous Rates of Change (Derivatives)

$$f'(a) := \lim_{\Delta x \to 0} \left(\frac{f(a+\Delta x) - f(a)}{\Delta x} \right)$$

derivative notation

Distance fallen in time t (by gravity)

$$s(t) = 16t^2$$

$$\frac{s(3) - s(0)}{3} = \frac{s(3)}{3} = \frac{16 \cdot 3^2}{3} = 48 \text{ fps}$$

$s(0)$ ← 0

instant velocity

$$v(t) = \lim_{\Delta t \to 0} \left(\frac{s(t+\Delta t) - s(t)}{\Delta t} \right)$$

$$= \lim_{\Delta t \to 0} \left(\frac{16(t+\Delta t)^2 - 16t^2}{\Delta t} \right)$$

$$v(t) = 16 \lim_{\Delta t \to 0} \left(\frac{\cancel{t^2} + 2t\Delta t + (\Delta t)^2 - \cancel{t^2}}{\Delta t} \right)$$

$$= 16 \lim_{\Delta t \to 0} \left(2t + \Delta t \right) = 16 \cdot 2t = 32t$$

So... speed formula is $v(t) = 32t$

$a(t) = v'(t) = 32 \ \underline{fps^2}$